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Predicting strength of fibrous laminates under triaxial loads only upon independently measured constituent properties

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ABSTRACT

A long standing and very challenging problem is to predict the ultimate strength of a fibrous laminate under arbitrary load condition only based on the mechanical properties of its constituents measured independently. Although the Bridging Model is unique for calculating the internal stresses in the constituent fiber and resin (which stands for a matrix material throughout this paper) materials subjected to any load including a temperature variation, the in situ mechanical properties of the constituents must be provided beforehand. A unidirectional (UD) composite exhibits a transverse tensile strength smaller than the tensile strength of the monolithic resin material, indicating that the in situ tensile strength of the resin in the transverse direction is different from that measured using monolithic material specimens. This is attributed to a stress concentration. The stress concentration factors (SCFs) of the resin material in a RVE (representative volume element) due to occurrence of the fiber are determined in terms of elasticity theory. The resin in situ tensile, compressive, and shear strengths in the transverse plane are obtained by the corresponding resin strengths measured independently divided by the respective SCFs, whereas the resin in situ longitudinal strengths together with all the other constituent properties are the same as their original counterparts. Using these originally provided constituent properties as input data, the Bridging Model has been applied to analyze the second World-Wide Failure Exercise (WWFE-II) problems. The model's predictions for all the problems have been compared with available experimental data. Favorable correlation has been found.

1. Introduction

It has been a long-term dream in the composite community that the mechanical properties including ultimate strength of a fibrous laminate subjected to an arbitrary load condition are estimated with reasonable accuracy only based upon an established constituent material database and all the geometrical parameters of the constituents without doing any experiment on the laminate or any of its consisting laminae [1]. Once this dream becomes a reality, the design and development of a new composite structure will be much easier and more efficient and expanded use of composites can be expected. To achieve this, a necessary condition is to know the internal stresses generated in the constituent fiber and resin materials of the composites at every load level. According to the assessment of the WWFE-I (the first World-Wide Failure Exercise), the Bridging Model established by the author was the only theory attended the exercise which was able to calculate the thermal stresses in the constituents due to a temperature variation [2].

However, the use of the Bridging Model for laminate strength prediction is not sufficient, as the model itself is only capable of describing the constitutive relationship of a UD lamina up to failure and of calculating its internal stresses in the constituent fiber and resin materials due to the application of an external load. To determine a lamina failure micromechanically, some stress failure criteria for the fiber and resin must be assigned. As can be understood, different failure criteria can result in different predicted strengths for the lamina even though the stress evaluations for the fiber and resin are accurate. More other issues must be resolved before a laminate failure analysis and ultimate strength prediction can be accomplished. These include, among others, a criterion for detecting an ultimate failure of the laminate and a stiffness discount scheme for a failed lamina if this failure is not correspondent to an ultimate failure. The most important is that the constituent in situ mechanical properties must be provided as some of them can be different from those measured independently.

It was found that the application of the Bridging Model in its early stage to predictions for the WWFE-I problems was only moderate in correlation with the experiments, although some better than the performances of the two other micromechanics models took part in the exercise [3]. There were two reasons why

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the predictions were not very high in accuracy. One was that the last-ply failure was defined as an ultimate failure, and another was that a total stiffness discount scheme was employed for any failed lamina no matter whether that failure was caused by a fiber or a resin failure. A significant advancement was made in Ref. [4], by incorporating a new ultimate failure criterion, a partial stiffness discount scheme, and a pure resin interlayer into two adjacent primary lamina layers. The predicted accuracy for the WWFE-I problems was improved a great deal, with a total score attained even higher than that obtained by any other phenomenological strength theory assessed in the exercise [4]. Unfortunately, the constituent in situ properties, mainly the resin in situ strength parameters, were still determined by retrieving from the strength data of the UD laminas. This has blinded the key advantage of applying the Bridging Model, namely to predict composite failure and ultimate strength without doing any experiment on the composite. Only after all the in situ mechanical properties of the constituents are defined upon independently measured property data, can the key advantage of the Bridging Model be realized.

Whereas all the elastic–plastic property parameters of the fiber, if any, and the resin required by the application of the Bridging Model can be taken to be those measured independently, the resin in situ transverse tensile strength must be smaller than its original counterpart, as a UD lamina exhibits a transverse tensile strength smaller than the resin tensile strength measured using monolithic material specimens. It is well known that there occurs a stress concentration when a plate of an isotropic material contains a circular hole. The in-plane tensile strength of the plate containing the hole is only one-third of its original strength. Similarly, there must be a stress concentration in the resin when a circular fiber cylinder is embedded in it. In terms of the stress fields obtained on elasticity theory, the stress concentration factors (SCFs) corresponding to transverse normal and shear stresses are determined. The resin in situ transverse strengths (tensile, compressive, and shear strengths) can be defined by the measured counterparts using monolithic resin specimens divided by the SCFs.

In this work, the WWFE-II problems have been analyzed by virtue of the Bridging Model only using the constituent mechanical properties originally provided by the organizers [5]. The model's predictions have been compared with available experimental data. Good correlation has been found between the predictions and the experiments. Detailed analyzing procedures are described in the subsequent sections.

2. Summary of the Bridging Model

To make this presentation self-contained, the bridging model theory is briefly summarized in this section. For more details, refer to our monograph [6].

2.1. Model development

Suppose a RVE (Fig. 1) of a UD composite is subjected to any stress state $\{\sigma_i\}^T=\{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\}$. Using an incremental approach, the stress increments generated in the resin and the fiber can be correlated by a non-singular matrix, called a bridging matrix, through

$$\{d\sigma_i^m\} = [A_{ij}]\{d\sigma_j^f\} \tag{1}$$

Making use of two fundamental homogenized equations for the RVE, i.e.,

$$\{d\sigma_i\} = V_f\{d\sigma_i^f\} + V_m\{d\sigma_i^m\}, \tag{2}$$

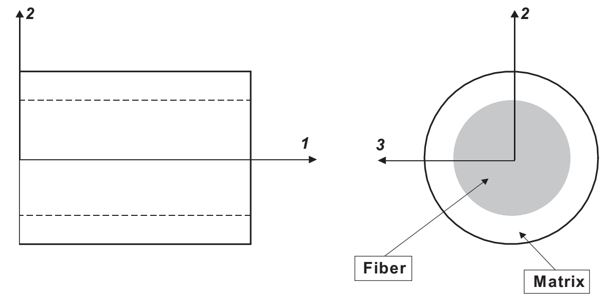

Fig. 1. A RVE (representative volume element) of a UD composite.

$$\{d\varepsilon_i\} = V_f\{d\varepsilon_i^f\} + V_m\{d\varepsilon_i^m\}, \tag{3}$$

where V_f and V_m are the fiber and resin volume fractions with understanding that $V_f + V_m = 1$ and $\{d\varepsilon_i\}=\{d\varepsilon_{11}, d\varepsilon_{22}, d\varepsilon_{33}, 2d\varepsilon_{23}, 2d\varepsilon_{13}, 2d\varepsilon_{12}\}^T$, together with the constitutive equations for the fiber, resin, and the composite, i.e,

$$\{d\varepsilon_i^f\} = [S_{ij}^f]\{d\sigma_j^f\} \tag{4.1}$$

$$\{d\varepsilon_i^m\} = [S_{ij}^m]\{d\sigma_j^m\} \tag{4.2}$$

$$\{d\varepsilon_i\} = [S_{ij}]\{d\sigma_j\} \tag{4.3}$$

where $[S_{ij}^f]$ and $[S_{ij}^m]$ are the compliance matrices of the fiber and the resin materials already known, one can easily obtain the following three fundamental equations of the Bridging Model:

$$\{d\sigma_i^f\} = (V_f[I] + V_m[A_{ij}])^{-1}\{d\sigma_j\} = [B_{ij}]\{d\sigma_j\}, \tag{5}$$

$$\{d\sigma_i^m\} = [A_{ij}](V_f[I] + V_m[A_{ij}])^{-1}\{d\sigma_j\}, \tag{6}$$

$$[S_{ij}] = (V_f[S_{ij}^f] + V_m[S_{ij}^m][A_{ij}])(V_f[I] + V_m[A_{ij}])^{-1}. \tag{7}$$

The total stresses in the fiber and resin at the current load level are simply updated through

$$\{\sigma_i^f\}_{k+1} = \{\sigma_i^f\}_k + \{d\sigma_i^f\}, \quad k = 1, \ldots, \tag{8.1}$$

$$\{\sigma_i^m\}_{k+1} = \{\sigma_i^m\}_k + \{d\sigma_i^m\}, \quad k = 1, \ldots, \tag{8.2}$$

whereas the stresses on the composite are given by

$$\{\sigma_i\}_{k+1} = \{\sigma_i\}_k + \{d\sigma_i\}, \quad k = 1, \ldots, \tag{8.3}$$

In Eqs. (5)–(7), $[I]$ is a unit matrix. Thus, the only quantity to be determined is the bridging matrix $[A_{ij}]$.

2.2. Characterization of the bridging matrix

The elements of the bridging matrix can be separated into dependent and independent. As a UD composite is transversely isotropic with only five independent material parameters, the bridging matrix $[A_{ij}]$ can only have five independent elements. Without any lost of generality and in light of the characteristics of the compliance matrix of the composite, let us take A_{11}, $A_{22}=A_{33}$, $A_{31}=A_{21}$, A_{32}, and $A_{55}=A_{66}$ to be independent. All the other elements are dependent, which should be determined according to the symmetric condition of the compliance matrix, i.e. (refer to Eq. (7))

$$S_{ji} = S_{ij}, \quad i,j = 1, 2, \ldots, 6 \tag{9}$$

It is noted that the element A_{44} is not independent, but should be obtained from the following equation for an elastic solution:

$$S_{44} = 1/G_{23} = 2(1+\nu_{23})/E_{22} = 2(S_{22} - S_{23}), \tag{10}$$