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Nonlinear bending analysis of ring-stiffened functionally graded circular plates under mechanical and thermal loadings

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ABSTRACT

The large deflection behaviors of ring-stiffened shear deformable functionally graded (FG) circular plates under mechanical and thermal loadings are investigated. Material properties of the FG plate are assumed to be temperature dependent, and graded through the thickness according to the power-law distribution of the volume fraction of the constituents. The nonlinear formulations are based on first-order shear deformation theory (FSDT) and the large deflection von Karman equations. In the theoretical model, the reaction of the stiffener on the plate is applied partly by means of body forces in the plate equilibrium equations. The force interaction is complemented with a set of plate–stiffener displacement compatibility equations. The dynamic relaxation (DR) method combined with the finite difference discretization technique is employed to solve the equilibrium equations. To verify the present solution, several examples are analyzed for linear/nonlinear bending of FG/isotropic circular plates with different boundary conditions. A detailed parametric study is carried out to investigate the influences of the material grading index, thickness-to-radius ratios, temperature dependency of material, temperature rise, stiffener depth, stiffener position and boundary conditions. Moreover, some linear and nonlinear analyses with different thickness-to-radius ratios are carried out to consider the effect of nonlinearity on the results.

1. Introduction

The use of stiffeners to enhance the transverse and/or in plane stiffness of flat plates is an efficient practice to achieve greater structural efficiency and material economy. Such plates may be treated as polar orthotropic when the stiffener arrangement is either radial, circumferential or a combination of the two and, on the condition that the number of stiffeners is large. To analyze these types of stiffened plates, research effort is focused on the transformation of the structural orthotropy to equivalent material orthotropy [2]. However, large numbers of stiffeners are usually avoided in practice, as this increases fabrication costs [1]. For practical and economic reasons, small numbers of stiffeners are generally used to stiffened plates [3]. Thus, the analysis of these stiffened plates which each stiffener treats as a discrete entity is of great importance. The design of the mentioned plate category is, however, more complicated. When only a few stiffeners are presented, the plate and stiffeners are treated as separate entities and kinematic continuity and force interaction at the plate–stiffener junctions is enforced [2]. Basu et al. [4] have developed a theory for analyzing the discretely stiffened rectangular plate. A discretely stiffened plate theory was developed for the elastic and elasto-plastic large deflection analysis of rectangular plates by Djahani [5]. In stark contrast to the situation for un-stiffened circular plates, the present state of knowledge of the behavior of stiffened circular plates is much less extensive [6]. Several years ago, Turvey [7] developed a discretely stiffened circular plate analysis and applied it to the elastic large deflection analysis of ring-stiffened circular composite plates. Subsequently, axisymmetric elastic and elasto-plastic large deflection versions of the discretely stiffened plate theory were developed and used to investigate the flexural response of ring-stiffened plates [8,9]. Turvey and Salehi [2,3] used a finite difference implementation of the dynamic relaxation (DR) algorithm to analyze non-axisymmetric elastic large deflection of solid and annular thin sector plates stiffened by a single eccentric rectangular cross-section radial stiffener. Using the similar method, Turvey and Salehi [1] also solved the governing equations for the elastic large deflection of uniformly pressure loaded axisymmetric discretely stiffened isotropic circular plates based on classical plate theory (CPT). The structural behavior of un-stiffened isotropic circular plates is now very well understood, though new knowledge continues to accumulate, particularly in relation to stiffened/un-stiffened plates made of composite and functionally gradient materials (FGMs). The FGMs were initially designed in 1984 by a group of material scientists in Japan, as thermal barrier materials for aerospace structural applications and fusion reactors [10]

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nowadays are developed for a more general use as structural components in extremely high-temperature environments. FGMs have continuously and smoothly varying material composition from one surface to another surface by gradually varying the volume fraction of constituent materials which lead to elimination of interface and layer problems [11]. Because of the advantages of FGMs over conventional composites and monolithic materials, these materials have been extensively studied for potential applications as structural elements, such as FGM beams, plates, shells, and cylinders [12–16]. Recently, Golmakani and Kadkhodayan [17–19], using the dynamic relaxation (DR) numerical method together with the finite difference discretization technique, gave a nonlinear bending analysis of moderately thick circular and annular functionally graded plates under mechanical and thermo-mechanical loading. To the knowledge of the author, there is no literature considering the nonlinear bending response of an axisymmetric ring-stiffened circular functionally graded plate under mechanical and thermo-mechanical loading. Hence, the present paper is concerned with a further development of the discretely ring-stiffened isotropic circular plate analysis used in Ref. [20], for the thermoelastic large deflection response of shear deformable FG circular plates with discrete ring stiffener based on the first-order shear deformation plate theory (FSDT). The FSDT is used to account for the transverse shear strains through the plate thickness. The material properties are assumed to be nonlinear functions of temperature and graded in the thickness direction according to the power-law distribution in terms of the volume fractions of the constituents. The plate is subjected to a uniform pressure loading in thermal environments and the boundary conditions are clamped and simply supported. The constitutive equations were obtained based on FSDT using the von Karman theory for large deflections. In the theoretical model, the reactions of the stiffener on the plate are applied partly by means of body forces in the plate equilibrium equations. The force interaction is complemented with a set of plate–stiffener displacement compatibility equations. The set of nonlinear equilibrium equations are converted into DR format and the finite difference discretization technique is applied to solve the plate governing equations. The limiting process technique is used to eliminate the singularity at the center of circular plate. The accuracy of the present results is examined by small and large deflection analyses comparison with those reported in [20]. Finally, a detailed parametric study is carried out to investigate the influences of the material grading index, thickness-to-radius ratios, temperature dependency of material, temperature rise, stiffener depth, stiffener position and boundary conditions on the large deflection behavior of the shear deformable ring-stiffened FG circular plates subjected to mechanical and thermo-mechanical loading. Also some linear and nonlinear analyses with different thickness-to-radius ratios are carried out to consider the effect of nonlinearity on the results.

2. Grading relation of material

It is assumed that the plate and stiffener are made of FGMs and isotropic elastic material, respectively. Generally, FGMs are heterogeneous composite materials so that these materials are nonhomogeneous in terms of both the material properties and microstructures. Typically these materials are made of a mixture of ceramic and metal in which the ceramic constituent of the material provides the high-temperature resistance and protects the metal from corrosion and oxidation. On the other hand, the FGM is toughened and strengthened by the metallic composition. Some models in the literature describe the variation in the mechanical and thermal properties in the FGMs. The most commonly used model is the power law distribution of the volume fraction. According to this model, the effective material properties S can be expressed as

$$S = S_c V_c + S_m V_m, \tag{1}$$

where subscripts m and c denote the metallic and ceramic constituents, respectively; V_c and V_m are the ceramic and metal volume fractions, respectively, and are related by

$$V_c + V_m = 1. \tag{2}$$

The ceramic volume fraction V_c is assumed to follow a power law distribution as

$$V_c = \left(\frac{2z+h}{2h}\right)^n, \tag{3}$$

where z is the thickness coordinate $(-h/2 \le z \le h/2)$ and the composition is considered to vary from upper to the lower surface of the plate such that the top surface $(z = h/2)$ is ceramic-rich, whereas the bottom surface $(z = -h/2)$ is metal-rich. For FGMs in high working temperature, significant variations in thermal and mechanical properties of the materials are expected [12]. For example, Young's modulus of stainless steel, nickel, Ti–6Al–4V and zirconia are reduced by 37, 21, 34 and 31%, respectively, when the temperature increases from room temperature 300 K to 1000 K [21]. Therefore, accurate calculation of the mechanical response needs accounting for this temperature dependency. The material properties S that are temperature dependent can be written as [22]

$$S = S_0(S_{-1}T^{-1} + 1 + S_1 T + S_2 T^2 + S_3 T^3), \tag{4}$$

where $T = T_0 + \Delta T$ and $T_0 = 300$ K (room temperature), S_0, S_{-1}, S_1, S_2 and S_3 are the coefficients of temperature T(K) and are unique for the constituent materials. Here, Young's modulus E and the thermal coefficient of expansion α are considered as temperature dependent (TD) material properties. The temperature variation is assumed to occur in the thickness direction only, and the temperature field is considered constant in the plane of the plate. For the mentioned one-dimensional temperature field, it is assumed that the upper ceramic surface is exposed to higher temperatures compared to the lower metal surface. In this case, the temperature distribution along the thickness can be obtained by the one-dimensional Fourier equation of heat conduction

$$-\frac{d}{dz}\left(k(z)\frac{dT}{dz}\right) = 0, \tag{5}$$

where $T = T_c$ at $z = h/2$ and $T = T_m$ at $z = -h/2$. As indicated in the work by Miyamoto [23], the temperature dependency of the heat conductivity coefficient k does not have a significant effect on results. Hence, the mentioned coefficient is assumed to be temperature independent (TID) in this study

$$k(z) = (k_c - k_m)\left(\frac{2z+h}{2h}\right)^n + k_m, \tag{6}$$

It is easy to compute the temperature function $T(z)$ from Eq. (7) as follows [24]:

$$T(z) = T_m + (T_c - T_m)\int_{-h/2}^{z}\frac{dz}{k(z)}\Bigg/\int_{-h/2}^{h/2}\frac{dz}{k(z)}. \tag{7}$$

It is notable that the integrations are computed numerically by discretizing the plate along the thickness direction. Consequently, E and α are both temperature and position dependent as follows:

$$\begin{cases} E(z,T) = (E_c(T) - E_m(T))\left(\frac{2z+h}{2h}\right)^n + E_m(T), \\ \alpha(z,T) = (\alpha_c(T) - \alpha_m(T))\left(\frac{2z+h}{2h}\right)^n + \alpha_m(T). \end{cases} \tag{8}$$

Generally, Poisson's ratio ν does not depend on the temperature change significantly and varies in a small range. Hence, for the sake of simplicity it is assumed to be a constant [25].