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Acoustic metamaterial beams based on multi-frequency vibration absorbers

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ABSTRACT

Presented here is a new metamaterial beam based on multi-frequency vibration absorbers for broadband vibration absorption. The proposed metamaterial beam consists of a uniform isotropic beam and small two-mass spring-mass-damper subsystems at many locations along the beam to act as multi-frequency vibration absorbers. For an infinite metamaterial beam, governing equations of a unit cell are derived using the extended Hamilton principle. The existence of two stopbands is demonstrated using a model based on averaging material properties over a cell length and a model based on finite element modeling and the Bloch–Floquet theory for periodic structures. For a finite metamaterial beam, because these two idealized models cannot be used for finite beams and/or elastic waves having short wavelengths, a finite-element method is used for detailed modeling and analysis. The concepts of negative effective mass and effective stiffness and how the spring-mass-damper subsystem creates two stopbands are explained in detail. Numerical simulations reveal that the actual working mechanism of the proposed metamaterial beam is based on the concept of conventional mechanical vibration absorbers. For an incoming wave with a frequency in one of the two stopbands, the absorbers are excited to vibrate in their optical modes to create shear forces to straighten the beam and stop the wave propagation. For an incoming wave with a frequency outside of but between the two stopbands, it can be efficiently damped out by the damper with the second mass of each absorber. Hence, the two stopbands are connected into a wide stopband. Numerical examples validate the concept and show that the structure's boundary conditions do not have significant influence on the absorption of high-frequency waves. However, for absorption of low-frequency waves, the structure's boundary conditions and resonance frequencies and the location and spatial distribution of absorbers need to be considered in design, and it is better to use heavier masses for absorbers.

1. Introduction

Metamaterials are a new class of semi-active composites proposed in 2001 by the Defense Advanced Research Projects Agency (DARPA) to exhibit exceptional material properties not observed in nature or in the constituent materials in order to develop new technologies [1–3]. The transcendental properties of metamaterials are assumed to arise from their engineered constructs using artificially fabricated, extrinsic, low dimensional inhomogeneities. This concept motivates engineers to dream and think out of the constraints imposed by the performance limitations of conventional materials. Metamaterials were first introduced for dealing with electromagnetic waves [1–7], and later researchers began to look into metamaterials for dealing with acoustic waves [8–14].

When an electromagnetic wave enters a material, its electric and magnetic fields interact with the material's electrons and other charges of atoms and molecules. This interaction alters the wave's speed and wavelength, especially when local optical resonance happens. Hence, it is possible to use this electromagnetic interaction to design a material with negative magnetic permeability (μ) and electric permittivity (ε) and hence a negative refractive index (n) [4]. Electromagnetic metamaterials are mainly designed by using the effects of negative refractive indices, cloaking, and superlensing, which are in turn caused by designed optical resonance. Possible applications include optical fibers with no transmission loss, very thin optical lenses, compact radar lenses with relatively aberration free performance, electromagnetic absorbers, technologies for rendering objects invisible, optical microscopes capable of observing atoms, subwavelength waveguides, photon tunneling, smaller antenna, backward wave antennae, artificial magnetic device composed of non-magnetic materials, high-performance permanent magnets, photolithography and nanolithography to make higher density electronic circuits, faster fiber-optic communications, detailed biomedical

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imaging in real-time and in vivo, and visualizing proteins in intact cells using optical microscopes instead of a more complicated method like X-ray crystallography [1–3]. For example, current optical microscopes can only resolve objects down to the 400 nm limit of visible light, which is about one tenth the diameter of a red blood cell. In contrast, optical superlens having negative refractive indices can image nano-scale structures with a resolution that is about one sixth the wavelength of visible light [15].

Recently, the analogy between electromagnetic waves and acoustic waves stimulates development of metamaterials for dealing with mechanical waves [8–14]. For electromagnetic waves having wavelengths much longer than the distance between atoms, a material is well described by its locally averaged electromagnetic properties ε (permittivity) and μ (permeability). Similarly, for elastic waves having wavelengths much longer than the distance between atoms, a material can be treated as a continuum and is well described by its locally averaged mechanical properties E (Young's modulus) and ρ (mass density). The transcendental properties (e.g., $\mu<0, n<0$ or $\mu \gg 1$) of an electromagnetic metamaterial are mainly caused by the optical resonance between an electromagnetic wave and the material's manmade subunits. Similarly, acoustic metamaterials can be designed by building mechanical subunits into a natural material to resonate with mechanical waves propagating in it. Similar to the interaction between an electromagnetic wave and a material's charges, an elastic wave in a structure may resonate with the structure's subunits and its speed and wavelength can be changed. This local mechanical resonance can be used to design acoustic metamaterials with dynamics-dependent negative effective mass and stiffness [11–14,16], and this is essentially a concept for designing semi-active composite materials.

Fortunately, some difficulties in designing optical metamaterials do not exist in designing acoustic metamaterials. For example, to achieve strong contrast in optical properties generally requires the use of metals, which are accompanied by strong and undesirable dissipation of energy. On the other hand, a combination of strong contrast and low energy dissipation can be easily achieved in acoustics and elastodynamics. Moreover, fabrication of acoustic metamaterials can be much easier than that of their optical counterparts. However, there are natural materials with negative electric permittivity for direct design of electromagnetic metamaterials, but, unfortunately, there are no natural materials with negative mass density or Young's modulus. Hence, acoustic metamaterials can be realized only by manmade small-scale composite structures.

Acoustic metamaterials are designed by building subunits to create the desired, unnatural material properties to manipulate the dispersive properties of elastic waves, and they are often bandgap periodic structures [17]. Unfortunately, modeling of metamaterials is challenging because they are complicated built-up composite structures. In the literature, only few simple models for bar-like acoustic metamaterials were proposed, and they are all based on spatial averaging of heterogeneous material properties over each subunit and hence are valid only for waves of wavelengths much longer than the sizes of subunits [8–12,16]. If a metamaterial is to behave like a homogeneous material described by its averaged material properties, its subunits must be much smaller than the shortest wavelength of waves propagating in it. The averaged dynamic material properties make a metamaterial made of non-dispersive materials, behaves like a dispersive one and cause the existence of a useful but mysterious phononic stopband that allows no waves within that frequency range to propagate forward [16,17]. Most current designs of acoustic metamaterials are based on the stopband effect [12–14]. To manufacture such metamaterials with tiny subunits in order to have stopbands, expensive manufacturing techniques are required, including micro- and nano-manufacturing techniques that are still under development.

Similar to optical (or electromagnetic) metamaterials, acoustic metamaterials enable development of many new technologies, including acoustic absorbers using the stopband effect, acoustic imaging below the diffraction limit using the superlensing effect, super environmental acoustic absorbers, and subwavelength waveguides. However, key challenges are the theoretical development of new metamaterials with different subsystems, physics-based modeling and understanding of working mechanisms, and experimental verification of such mechanics-based, acoustic metamaterials.

Our previous studies [18–20] reveal that the actual working mechanism of acoustic metamaterials is based on the concept of conventional vibration absorbers. A conventional vibration absorber consists of a lumped mass attached to the main mechanical system by a linear spring and hence has only one local resonance frequency. A conventional vibration absorber uses the 1:1 external resonance between the forcing frequency on the main system and the local resonance frequency of the absorber to transfer the vibration energy to the absorber and stop the main system's motion. The concept of conventional vibration absorbers can be extended to use the n:1 ($n=2,3$ or other larger integers) external resonance between the forcing frequency on the main system and the local resonance frequency of the absorber [21–24]. However, the main system needs to be coupled with the absorber by nonlinear quadratic (or cubic or other higher-order) terms. The unique feature of this nonlinear vibration absorber is that the vibration energy transferred to the absorber will remain in the absorber (or be dissipated inside the absorber) without reentering the main system even after the excitation on the main system stops. Hence, the transient vibration time will be short after a transient excitation on the system. Unfortunately, a nonlinear vibration absorber (especially when high-order nonlinear terms are used) takes a transient time longer than that of the conventional 1:1 vibration absorber to develop the absorber's oscillation amplitude to absorb the vibration energy. Hence, nonlinear absorbers may not even work for short-time transient excitations, and 1:1 linear absorbers are worth more studies for further improvement.

Previous research results [18–20] indicate that, for an acoustic metamaterial with single-mass absorbers, a stopband exists at the high-frequency side of the local resonance frequency of each absorber. Our hypothesis here is that, if each vibration absorber has two lumped masses and hence two local resonance frequencies, there will be two stopbands. If the two local resonance frequencies are designed to be close to each other, the two stopbands can be combined into a wide stopband. This paper is to perform theoretical development and numerical validation for this new metamaterial for broadband absorption of transverse elastic waves in beams. The specific objectives are: (1) to develop appropriate modeling and analysis methods and demonstrate absorption of elastic waves in a beam with integrated spring-mass subsystems having two local resonance frequencies (e.g., Fig. 1), (2) to reveal the actual working mechanism of this metamaterial beam for absorption of low- and

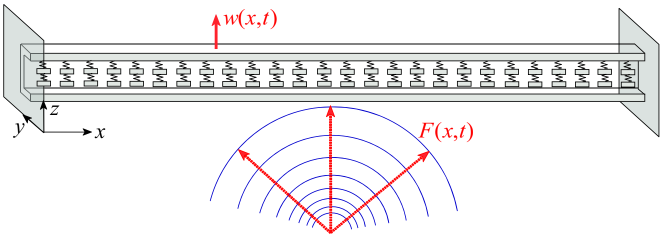

Fig. 1. A multi-frequency metamaterial beam for vibration absorption.