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Free vibration analysis of Mindlin plates partially resting on Pasternak foundation

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ABSTRACT

In this paper, the generalized differential quadrature (GDQ) method is used to study free vibration of moderately thick rectangular plate partially resting on Pasternak foundation. The foundation is considered to support the plate either completely or partially. The governing equations which consist of a system of partial differential equations (PDEs) are obtained based on the first-order shear deformation theory. Various combinations of simply supported, clamped and free boundary conditions are considered. Application of the GDQ method to the governing PDEs resulted in a system of algebraic equations. Solution of this system with accordance to the considered boundary conditions leads to an eigenvalue problem to obtain natural frequencies of the plate. Results of this study are validated with available results in the literature which reveal accuracy and fast convergence rate of the method. Effects of different parameters such as foundation stiffness, foundation geometry, boundary conditions and geometrical parameters on the natural frequencies of the plate are presented.

1. Introduction

Rectangular plates are one of the important design elements in different branches of modern engineering fields including mechanical, marine, aerospace, optical and structural engineering. Thus, vibration and modal analysis of such plates have been a main subject of research studies for a long period [1,2]. The governing equations for plate deformation have been obtained by several methods. Among earlier and simple theories, one may refer to the classical plate theory, also known as Kirchhoff plate theory; which results in erroneous predictions as thickness of the plate increases. In order to achieve more accurate predictions Mindlin [3] and Reissner [4] considered the effect of transverse shear deformation and rotary inertia in their plate models. This assumption adds two rotational degrees of freedom to the model which allow constant transverse shear strain distribution through the thickness. Due to the constant transverse shear strain hypothesis, this theory is also called the First-order Shear Deformation Theory (FSDT) and a shear correction factor is demanded to reduce the errors [4]. Higher-order shear deformation theories (HSDT) were also proposed by Wang et al. [5] for analysis of different mechanical structures; which was subsequently used by Simsek and Kocatürk [6] and Viola et al. [7] for vibration analysis of beams, shells and panels. However, the First-order Shear deformation theory is still common for analysis of moderately thick plates.

Using FSDT, the governing equations of plates would be a system of three coupled partial differential equations. This system of equations cannot be solved analytically except for some special cases of boundary conditions, e.g. simply supported at all or at least opposite edges. Thus, numerical methods, as powerful alternatives, have been used to obtain results for different types of boundary and loading conditions. Among these numeric techniques, Rayleigh–Ritz in particular received a lot of attention, see for instance [8–10], who adopted two dimensional and Gram–Schmidt polynomials as the admissible functions while Shen et al. [11] introduced a new set of admissible functions to analyze plates with four free edges. Malekzadeh et al. [12] and Liu and Liew [13], also, proposed a solution based on differential quadrature element method formulation which was also implemented by many other researchers.

The vibration analysis of moderately thick plates on various types of elastic foundation has also been attracted by many researchers. Various engineering systems such as footing of buildings, pavement of roads and foundation of heavy machines can be considered as examples of thick plates on elastic foundations. The mechanical behavior of foundation was widely discussed by Winkler [14] and Pasternak [15].

Different investigations have been done in order to describe vibration of rectangular plates on elastic foundation in which the Pasternak model was widely applied. The exact solution of bending,

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buckling and vibration of Levy-type plates resting on elastic foundation was derived by Lam et al. [16]. The analytical solution for vibration of Mindlin plates on elastic foundation was derived by Xiang et al. [17] which was confined to simply supported plates. Omurtag et al. [18] implemented the finite element method to study vibrational behavior of rectangular plates on elastic foundation with different boundary conditions.

It is interesting to note that nearly all previous studies in the literature are restricted to plates completely resting on Pasternak foundation while plates with partial supports received much less attention. The partially supported plates, e.g. plates with foundation strips along their edges or partial rectangular support parallel to their edge may appear in many engineering structures, such as; plates used to cover temporary holes in a street, offshore platforms with local supports or cavities in various structures. As examples of previous studies on partially supported plates, one may refer to the study by Motaghian et al. [19] in which the free vibration response of thin isotropic rectangular plates with partial supports for different combinations of clamped and simply supported edges is presented. Xiang [20] studied the vibration of rectangular Mindlin plates resting on non-homogenous elastic foundations which is again restricted to the plates with at least two opposite simply supported edges and special type of foundation. Recently, the static analysis of plates resting on boundary strips was investigated by Nobakhti and Aghdam [21]. However, studies related to free vibration of moderately thick plates partially resting on Pasternak foundations with different types of boundary conditions are not found in the open literature.

In this study, the GDQ method is used to study free vibration analysis of moderately thick plates partially resting on Pasternak foundation. The governing equations are derived based on the first-order shear deformation theory. Plates with different combinations of free, simply supported and clamped edges are considered. Numerical solutions are developed through solving the governing differential equations of plates. Results are discussed in detail through parametric and verification studies. It was concluded that proposed numerical solution can successfully predict the natural frequencies of plates on partial elastic foundations.

2. Governing equations

A rectangular isotropic plate with length a, width b and thickness h is considered. The plate is partially resting on Pasternak type foundation with two different geometries. The first geometry consists of boundary strips elastic foundations along edges, with width t_{x1} and t_{x2} along the x-direction and t_{y1} and t_{y2} along the y-direction as shown in Fig. 1(a). The second geometry for partially supported foundation includes a rectangle foundation parallel to the plate which does not essentially cover the whole plate as illustrated in Fig. 1(b). The next geometry of foundation is a rectangular plate resting on elastic supports at its corners as illustrated in Fig. 1(c). The first case might be interested in plates covering holes in streets or structures while the two later cases are of particular interest due to their application in various offshore platforms.

The undeformed middle surface contains the x and y axes of the Cartesian coordinate system (x, y, z). According to Mindlin Plate Theory, three fundamental variables are used to express the displacements in x, y and z directions, which are

$$u=-z\psi_x(x,\ y,\ z,t) \tag{1a}$$

$$v=-z\psi_y(x,\ y,\ z,t) \tag{1b}$$

$$w=w(x,\ y,\ z,t) \tag{1c}$$

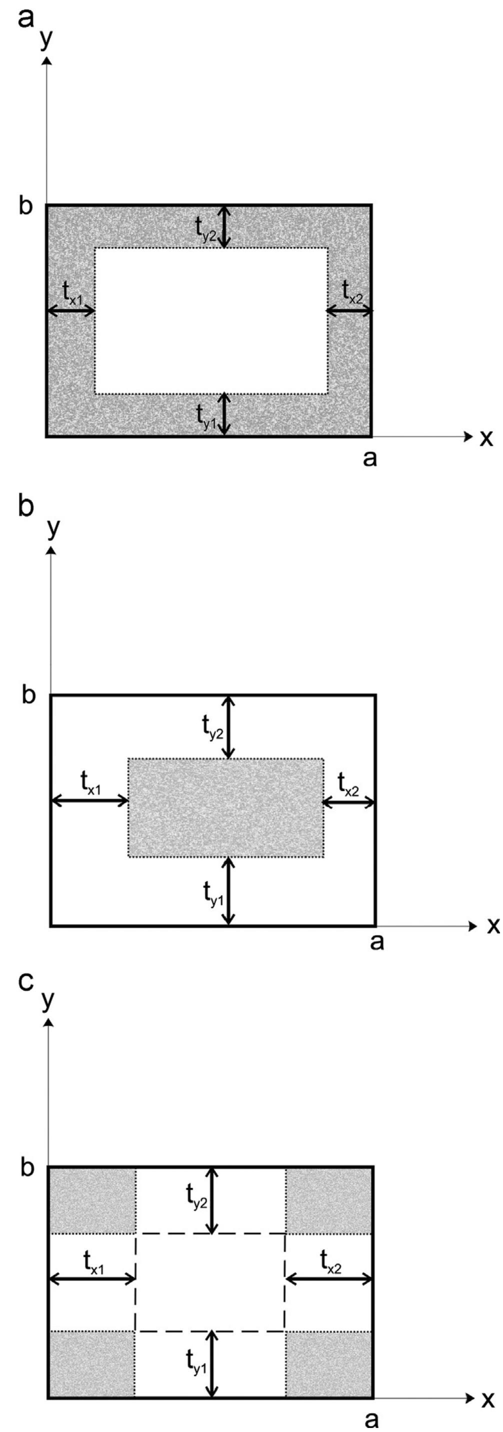

Fig. 1. (a) Partially supported plate with boundary strips along edges. (b) Partially supported plate with rectangular foundation parallel to the plate edges. (c) Partially supported plate with elastic foundation at the corners of the plate.

where t is the time, w denotes transverse deflection of the middle surface of the plate, ψ_x and ψ_y are the rotations of a normal line due to plate bending.

In order to derive the governing equations, the following dimensionless parameters are defined as

$$X=\frac{x}{a};\ Y=\frac{y}{b};\ \mathbf{W}=\frac{w}{h};\mathbf{\Psi}_X=\psi_X;\mathbf{\Psi}_Y=\psi_Y;$$

$$\beta=\frac{a}{b};\ \gamma=\frac{h}{b};\ \delta=\frac{h}{a};\ Q=\frac{q}{\kappa Gh};\ \alpha=\frac{6\kappa(1-\nu)}{\delta^2};$$