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Stress analysis in symmetric composite laminates subjected to shearing loads

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ABSTRACT

In the present study, an analytical solution is proposed to determine the interlaminar stresses in long symmetric laminated composite plates subjected to shearing loads. An improved first-order shear deformation theory (IFSDT) and a simplified IFSDT (SIFSDT) are utilized to calculate the unknown constants appearing in the reduced elasticity displacement field which actually demonstrate the global deformations of the plate. Then the numerical values of the interlaminar normal and shear stresses are established through the laminates by using Reddy's layerwise theory. Finally, several numerical examples are presented to study the interlaminar stresses in the interfaces of the layers and through the thickness of the laminates. The effects of fiber orientation angles as well as the stacking sequence of the layers within the laminates on the interlaminar stresses are obtained and discussed. All results specify high stress gradients of the interlaminar normal and shear stresses near the edges of the laminates.

1. Introduction

1.1. Background

Existence of boundary-layer regions for interlaminar stress fields near the free edges of composite laminates is one of the most important problems in design and analysis of such materials. In the boundary-layer regions, due to geometry and material discontinuities through the thickness of the laminates, the interlaminar stresses can start to grow rapidly to reach the values much higher than those predicted by the classical lamination theory (CLT). These highly concentrated stresses can lead to destructive failures in the laminates such as delamination and transverse cracking. Because of complexity and locally three-dimensional nature of boundary-layer problem in the free edges of composite laminates, no exact elasticity solution has been developed to this problem so far, but several approximate analytical and numerical methods have been proposed in the last four decades. A complete literature review on these methods is available in survey articles of Kant and Swaminathan [1] and Mittelstedt and Becker [2].

1.2. Analytical methods

The first approximate analytical solution on interlaminar stresses in laminated composite plates belongs to Puppo and Evensen [3] who used anisotropic laminas with separating isotropic shear layers to model a finite-width composite laminate while neglecting the interlaminar normal stresses throughout the laminate. Later, other approximate analytical methods were developed to study the edge-effect problem in composite laminates. The most important ones are the use of simple stress approximations by Whitney [4], the approximate elasticity solutions by Pipes and Pagano [5], higher-order theories by Pagano [6], the boundary-layer theory by Tang and Levy [7], the perturbation technique by Hsu and Herakovich [8], and a global high-order shear deformation theory by Lo et al. [9]. By means of Reissner's variational principle, Pagano [10] developed an approximate analytical solution based on assumed in-plane stresses to study the stress fields within and near the free edges of composite laminates. Wang and Choi [11,12] used Lekhnitskii's stress functions and anisotropic theory of elasticity to study the singularity of stress fields at the boundary layers of a laminated composite plate. Using the force balance method along with the principal of minimum total complementary energy, Kassapoglou and Lagace [13] proposed an analytical solution to study the interlaminar stresses in a symmetric composite laminate under the uniaxial loading. Yin [14,15] employed the principal of minimum total potential energy and Lekhnitskii's stress functions to study the free-edge stresses in a long composite laminate under uniaxial extension, bending and

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twisting. With a complementary energy approach, Kim and Atluri [16] examined the interlaminar stresses in composite laminates under both mechanical and thermal loading. Cho and Kim [17] used the principal of complementary virtual work along with Kantrovich method to analyze the free-edge stresses in composite laminates under extension, bending, twisting and thermal loading. Recently, based on reduced elasticity displacement field, an improved first-order shear deformation theory (IFSDT) and Reddy's layerwise theory were employed by Yazdani Sarvestani [18] to determine the interlaminar stresses in long general composite laminates under extension and bending.

1.3. Numerical methods

Finite element, finite difference, and boundary element methods are the most important numerical methods have been used for the last four decades to study the free-edge stresses in laminated composite plates. Using finite difference methods, Pipes and Pagano [19] solved two-dimensional governing elasticity equations to determine the numerical results for interlaminar stresses near the free edges of a long symmetric angle-ply laminate under uniform extension. Later, Wang and Crossman [20] developed a quasi-three-dimensional finite element solution to determine the free-edge stresses in a symmetric balanced composite laminate under uniaxial extension and uniform thermal loading. Whitcomb et al. [21] studied the differences in numerical results for interlaminar stresses obtained by various methods (finite difference methods, finite element methods, and perturbation techniques). They showed that the finite element method is the most reliable method among the others. Boundary element method and the integral equation theory were used by Davi [22] to study the stresses in a general laminate under uniform axial strain. Using the Reissner mixed variational theorem (RMVT), Carrera and Demasi [23,24] examined the accuracy of the finite-element mixed layerwise solutions that they proposed, by comparing the numerical results for interlaminar stresses in several finite-element models and elasticity theory within composite laminates and sandwich plates. Nguyen and Caron [25] used a multi-particle finite element method to study the interlaminar stresses near the free edges of general composite laminates under mechanical and thermal loading. Other approximate numerical methods are a two-dimension to three-dimension global-local finite element method by Thomson and Griffin [26], using a layerwise laminate theory by Robbins and Reddy [27], and a three-dimensional multi-layer higher-order finite element method by Gaudenzi et al. [28].

1.4. Present study

According to the literature survey, no paper has been published so far on the edge-effect problem in composite laminates subjected to shearing loads. The objective of the present study is to propose an analytical solution to determine the interlaminar stress fields within and near the free edges of symmetric composite laminates subjected to a shearing load. The work begins with consideration of the most general elasticity displacement field in long laminated composite plates. Then by employing several assumptions regarding the physical behavior of the deformation of a long symmetric laminate, a reduced elasticity displacement field is obtained. There are two types of unknown parameters in the displacement field to be determined: global and local parameters. The global parameters belong to all layers and describe the global deformations of the laminate and can be determined by employing a suitable equivalent single-layer (ESL) theory. Here the improved first-order shear deformation theory (IFSDT) and the simplified IFSDT (SIFSDT) are used for this purpose due to their simplicity and effectiveness. On the other hand, the local parameters in the displacement field describe the local deformations within the laminate and must be determined by using a more accurate and reliable theory such as Reddy's LWT used in this study.

2. Theoretical formulation

An Nth-layered long symmetric composite laminate under a shearing load at $x=\pm a$ is considered here in Fig. 1. The coordinate system (x, y, z) is located at the middle plane of the laminate, with its length, width, and thickness being equal to $2a$, $2b$, and h, respectively. Here, it is assumed that the plate is long in the x-direction so that the strains away from the ends ($x=\pm a$) of the laminate can be assumed to be only functions of y and z. Therefore, the integrations of the three-dimensional elasticity strain–displacement relations [29] within the kth layer of the laminate will generate the most general form of displacement field which has been shown to be as what follows:

$$u_1^{(k)}(x,y,z)=B_4xy+B_6xz+B_2x+u^{(k)}(y,z)$$
$$u_2^{(k)}(x,y,z)=-B_1xz+B_3x-\frac{1}{2}B_4x^2+v^{(k)}(y,z)$$
$$u_3^{(k)}(x,y,z)=B_1xy+B_5x-\frac{1}{2}B_6x^2+w^{(k)}(y,z) \quad (1)$$

where $u_1^{(k)}(x,y,z)$, $u_2^{(k)}(x,y,z)$, and $u_3^{(k)}(x,y,z)$ are the components of the displacement field for a material point initially located at (x,y,z) in the x-, y-, and z-directions, respectively. Also B_i's $(i=1,2,...,6)$, $u^{(k)}$, $v^{(k)}$, and $w^{(k)}$ are some unknown constants and functions resulted from the integrations. It is next noted that if the loading conditions at $x=-a$ and $x=a$ are identical, the following conditions must be hold:

$$u_1^{(k)}(x,y,z)=-u_1^{(k)}(-x,-y,z)$$
$$u_2^{(k)}(x,y,z)=-u_2^{(k)}(-x,-y,z)$$
$$u_3^{(k)}(x,y,z)=u_3^{(k)}(-x,-y,z) \quad (2)$$

Upon imposing these conditions on Eq. (1), it is readily concluded that:

$$u^{(k)}(y,z)=-u^{(k)}(-y,z)$$
$$v^{(k)}(y,z)=-v^{(k)}(-y,z)$$
$$w^{(k)}(y,z)=w^{(k)}(-y,z) \quad (3)$$

and $B_4=B_5=0$ and the displacement field in (1) is, therefore, the most general form of an arbitrary laminated composite plate simplified to be as follows:

$$u_1^{(k)}(x,y,z)=B_2x+B_6xz+u^{(k)}(y,z) \quad (4a)$$

$$u_2^{(k)}(x,y,z)=-B_1xz+B_3x+v^{(k)}(y,z) \quad (4b)$$

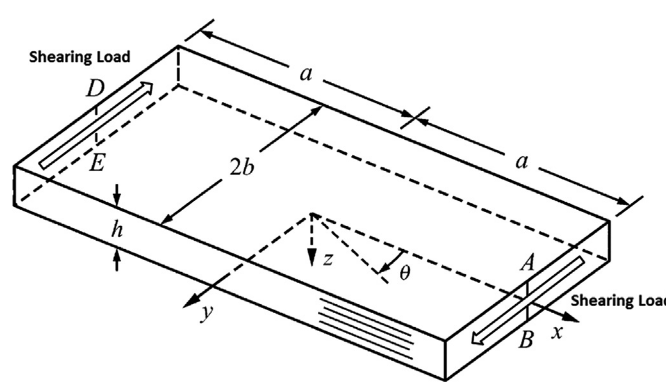

Fig. 1. Laminate geometry and coordinate system.