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A new shear deformation theory for the static analysis of laminated composite and sandwich plates

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ABSTRACT

In the present work, a new Inverse Trigonometric Zigzag Theory is proposed and implemented for the static analysis of laminated composite and sandwich plates. The theory assumes the higher order displacement field across the plate thickness satisfying the continuity conditions at the layer interfaces. Zero transverse shear stress boundary conditions at the top and bottom surfaces of the plate are also satisfied. An efficient C^0 finite element model is developed and employed to investigate the static response of laminated and sandwich plates. Numerical examples covering different features of laminated composite and sandwich plates are pronounced in the present study. The performance of the model is observed by comparing the evaluated results with different published results available in literature which ascertain its precision and range of applicability.

1. Introduction

Composites are widely applicable in aerospace, civil, mechanical and other fields of modern technology due to their favorable characteristics of high stiffness and strength to weight ratio. However, these materials carry complex configuration and are weak in shear due to its low shear modulus compared to extensional rigidity. Moreover, modeling of inter-laminar shear stresses becomes more critical in the layered laminated structures and is more complex in sandwich laminates as the material property variation is very large between the face and core layers. Therefore, the development of appropriate models capable of accurately predicting the behavior of these laminated structures is necessary. So, literature containing the investigations of different plate theories is carried out. In the early stage of the development of models, the classical laminate plate theory (CLPT) [1], which is an extension of Classical plate theory [2,3] has been employed to predict the mechanical behavior of composite structures. However, CLPT becomes inadequate for the analysis of multilayer composites by ignoring the effect of transverse shear deformation. To consider the transverse shear effect in plate bending, a number of first order shear deformation theories (FSDT) have been proposed. The FSDTs [4,5] assume linear stresses and displacements respectively through the laminate thickness. Yang et al. [6], Whitney [7], Whitney and Pagano [8], and Reissner [9–11] considered transverse shear effect using the FSDTs. However, the transverse shear strains in these theories remain constant along the thickness direction, so shear correction factors have to be required to adjust the transverse shear stresses at top and bottom surfaces. Therefore, the accuracy of the first-order theory directly depends on the reasonable shear correction factors and again the shear correction factors varies with the loading conditions, lamination sequence and boundary conditions as informed by Pai [12]. To overcome the shortcomings of CLPT and FSDT, various polynomial and non-polynomial higher order shear deformation theories (HSDT) are proposed, where the higher order variation of in-plane displacements through the thickness is considered to represent the actual warping of the plate cross-section making HSDT free from shear correction factor. The polynomial shear deformation theories consider a Taylor series expansion of higher order terms to represent the displacement vector, which was proposed and discussed by different investigators [13–22]. A number of HSDTs are also developed for analyzing functionally graded material [23–29]. Various researchers have developed a number of theories [30–39] considering the non-polynomial higher order theories where shear strain functions are taken into account for presenting the displacement field. The above discussed laminated plate theories are based on equivalent single layer approach in which the whole laminate is modelled as an equivalent single anisotropic layer. As shear correction factor is eliminated in HSDT, transverse shear strain gives continuous variation across the thickness which leads to discontinuity in the variation of the transverse shear stresses at the layer interfaces. But actually, the transverse shear stress is continuous at the interfaces whereas the strain may be discontinuous. It became the major drawback of HSDT. The development of the improved plate theories was initiated with the layer-wise (LW) plate theories to overcome the above disparity. Consequently, layer-wise

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models have been developed by Di Sciuva [40]. Lu and Liu [41], Robbins and Reddy [42] and many others considered a unique displacement field in each layer and displacement continuity across the layers taking unknowns at each layer interface. Mantari [43,44] and Roque et al. [45] modelled composite plates by trigonometric layerwise deformation theory. Ferreira [46] analyzed the composite plates considering layer-wise theory. The mixed layer wise models are also developed by Toledano and Murakami [47] and Carrera [48] which predicts inter-laminar stresses and displacements directly. Though these theories predict quite impressive results, they require significant computational involvement in analyzing a multi-layered plate as the number of unknowns increase with the number of layers.

The zigzag model came into account to overcome the drawbacks of layer-wise theories which satisfy the inter-laminar continuity conditions at all the interfaces making the number of unknowns independent of the layers. A major development in this direction is due to Di Sciuva [49], Murakami [50] and Liu and Li [51]. As an improvement to these theories, the traction free boundary condition at top and bottom of the plates is also satisfied. Bhasker and Varadan [52], Savitri and Varadan [53] and Lee et al. [54] have enlighten our path in this direction considering the variation of in-plane displacements to be a superposition of a piecewise linearly varying field on an overall higher order variation. Cho and Parmerter [55,56] presented a refined higher order shear deformation theory including piecewise cubic through thickness variation of the in-plane displacement with Heaviside step function. Di Sciuva [57,58] modified his own work [49] including piecewise cubic through thickness variation of the in-plane displacement which satisfies zero transverse shear stresses at top and bottom with continuity of these at layer interfaces. Carrera [59] presented a historical review on the zigzag theories used for the analysis of multilayered laminated plates and shells, in which the three basic theories have been discussed, namely: lekhnitskii multilayered theory, ambartsumian multilayered theory and reissner multilayered theory.

Besides the above zigzag theories, Carrera [60] and Demasi [61] considered higher order terms in the displacement field, using Murakami's zig-zag function and assumptions of transverse stresses brings about a large number of variables. However, applying static condensation technique allows to eliminate the unknowns related to the transverse stresses and thus, to derive efficient plate theories [62,63]. Furthermore, Rodrigues et al. [64] and Neves et al. [65] implemented a murakami's zigzag function to describe the structural kinematics of the multilayered plates.

Exact solutions for plates are only available under classical boundary conditions. Owing to limitations of the analytical methods, the finite element method (FEM) becomes one of the most popular numerical approach for analyzing the plate structures. Moreover, Di Sciuva [66,67], Cho and Parmerter [68], Chakrabarti and Sheikh [69], Kapuria and Kulkarni [70], Pandit et al. [71], Singh et al. [72], Chalak et al. [73] and Sahoo and Singh [74] focused on the zigzag theory with finite element representations of composite plates. Moreover, it maintains interlaminar continuity (IC) and zigzag requirement without increment of field variable with the increase in number of layers.

By reviewing the development of laminated plate theories, it is observed that, the zigzag theories that include the higher order polynomial terms predict the inter-laminar behavior accurately, at the cost of high computational efforts as the number of degrees of freedom are more. Therefore, in order to predict the inter-laminar behavior computationally efficient, minimum number of field variables should be taken into consideration ensuring that all the criteria are satisfied. Henceforth, it is necessary to develop a new displacement field which satisfies the traction free boundary conditions at top and bottom of the layers as well as inter-laminar continuity condition among each layer considering less number of unknowns. It can also be marked from the past reviews that no displacement field has been introduced so far in open literature where inverse trigonometric function has been implemented in zigzag theories and there is no FEM result published yet using the inverse trigonometric functions as shear strain functions combined with zigzag concept. Keeping all these viewpoints in mind, the present study deals with the development of a new theory and generalization of higher order zigzag theories which may be referred as a new Inverse Trigonometric Zigzag Theory. An inverse trigonometric function specifically inverse of cotangent function is considered as the shear strain shape function, which gives non-linear distribution of transverse shear stresses and zero transverse normal strain. Moreover, the proposed theory fulfills the inter-laminar continuity and zigzag requirement efficiently considering five number of field variables, as in the case of FSDT. It makes the displacement field computationally more efficient than the corresponding displacement field included with polynomial higher order terms. To implement this theory, a computationally efficient C^0 finite element is developed and applied to solve many sandwich plate problems and relatively simpler behavior of laminated composite plates with different boundary and loading conditions. Numerical results are evaluated for static response of laminated composite and sandwich plates in MATLAB environment. The accuracy and range of applicability of the present formulation are established by comparing the present results with 3D elasticity solutions and other existing shear deformation theories and the excellent result confirms the efficiency of the present model.

2. Mathematical treatments

The accurate analysis of any general structural problem considers the displacement field, constitutive relations and strain–displacement relations. A general laminated composite panel with dimensions $(a \times b \times h)$ in Cartesian co-ordinate system (***x–y–z***) is considered here as shown in Fig. 1. The plate is assumed to be constructed of arbitrary number of linearly, elastic orthotropic layers.

2.1. Displacement field approximations

In this section, a new inverse trigonometric zigzag theory (ITZZT) considering non-polynomial shear-strain functions is proposed for the composite plate. The theory considers the layer wise effect and inter-laminar continuity by the implementation of Heaviside step function named as inverse trigonometric zigzag theory. The through thickness variation of the in-plane displacements are assumed as a combination of a linear zigzag model with different slopes and the parameter $\Omega^{(k)}$ in each layer as shown in Fig. 2 and may be expressed as follows:

$$U_1(x,y,z) = u(x,y) - z\frac{\partial w}{\partial x} + \sum_{i=1}^{n_{u-1}} (z - z_i^u) H(z - z_i^u) \alpha_{xu}^i + \sum_{j=1}^{n_{l-1}} (z - z_j^l) H(-z + z_j^l) \alpha_{xl}^j + \left[g(z) + \Omega^{(k)} z \right] \beta_x$$

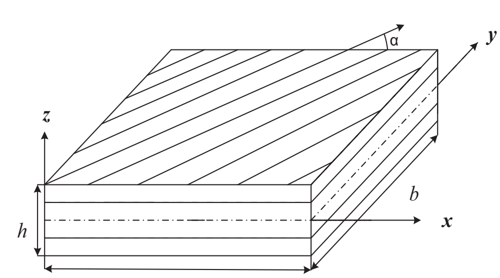

Fig. 1. Schematic diagram of a laminate.