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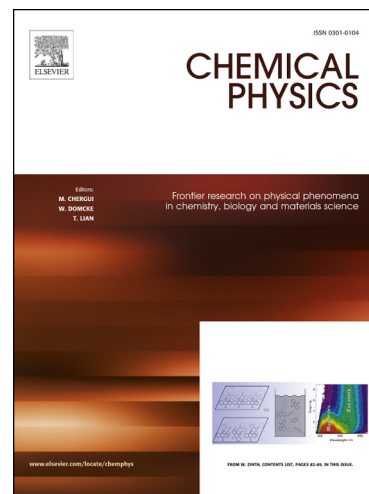
Shoichi Nagata

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An alternative expression to the Sackur-Tetrode entropy formula for a classical ideal gas

SHOICHI NAGATA

Department of Applied Sciences, Muroran Institute of Technology, 27-1 Mizumoto-cho, Muroran, Hokkaido, 050-8585, Japan

ABSTRACT

An expression for the entropy of a monatomic classical ideal gas is known as the Sackur-Tetrode equation. This pioneering investigation about 100 years ago incorporates quantum considerations. The purpose of this paper is to provide an alternative expression for the entropy in terms of the Heisenberg uncertainty relation. The analysis is made on the basis of fluctuation theory, for a canonical system in thermal equilibrium at temperature T . This new formula indicates manifestly that the entropy of macroscopic world is recognized as a measure of uncertainty in microscopic quantum world. The entropy in the Sackur-Tetrode equation can be re-interpreted from a different perspective viewpoint. The emphasis is on the connection between the entropy and the uncertainty relation in quantum consideration.

Keywords:

Sackur-Tetrode formula
Classical ideal gas entropy
Uncertainty relation
Fluctuation analysis

1. Introduction

The entropy of a monoatomic classical ideal gas has been given independently by the Sackur [1,2] and Tetrode [3,4], which is known as Sackur-Tetrode equation (ST-equation). This is a pioneering investigation about 100 years ago which incorporates quantum considerations. This entropy S at temperature T can be written as

$$\frac{S}{Nk_B} = \frac{5}{2} + \ln \left\{ \frac{V}{N} \left(\frac{2\pi mk_B T}{h^2} \right)^{3/2} \right\}, \quad (1)$$

which shows a fundamental step towards modern physics. Here N is the number of particles in the gas, k_B is Boltzmann's constant, V is the volume of the gas, m is the mass of a gas particle and h is Planck's constant. The detailed explanation and summary for the derivation of the ST-equation is described in a textbook [5].

Experimental verification and validity of this equation is not easy because the ideal gas does not exist indeed, and real gas becomes condensed state at low temperature. Experimental measurements and analyses are carried out for Ne, Ar, Kr and Hg, with a calorimetric method. Their vapors are monoatomic and behave in good approximation as ideal gases in gaseous phase. These experimental analyses, taking into account the corrections caused from the difference between the real monoatomic gases and the ideal gas, are described in detail in Ref. [6]. The experimental justification and correctness of the ST-equation has been inferred.

The almost forgotten episode for the physics of derivation of the ST-equation is highlighted in a recent paper "On the 100 th anniversary of Sackur-Tetrode equation" by W. Grimus [7] He emphasizes that the charm of the ST-equation has not changed over

100 years, which never fades away timelessly, and its importance in development of quantum theory and statistical mechanics.

What kind of re-interpretation for the ST-equation is possible? Is it really possible to add new explanation for entropy? This paper intends to provide an alternative expression for the entropy of a monoatomic classical ideal gas. Consequently, the entropy of a classical ideal gas can be written in terms of the thermal uncertainty relation based on the Heisenberg uncertainty relation. The key issue is to introduce the thermal uncertainty relation, which will be defined below.

2. General remarks on the Sackur-Tetrode equation and a possible extension

In the derivation of ST-equation, the following two conditions (a) and (b) are considered in the context the quantum nature.

(a) The entropy is obtained by the Boltzmann relation $S = k_B \ln W$, where W is the number of accessible microscopic states and k_B is the Boltzmann's constant. In order to count the number of microscopic possible states W , Sackur and Tetrode have introduced the elementary cell or domain in phase space of position and momentum. In this way, the phase space needs to be quantized in cells of volume h^f , here f is the number of degrees of the freedom of the system. The Planck's constant h is used properly, more than ten years before the establishment of quantum mechanics [7].

(b) The other assumption is made in deriving the ST-equation. The correct number of configurations in phase space is counted by taking into account the indistinguishability of atoms in the non-localized system for a gaseous state, N is the number of particles in the gas. The possible permutations $N!$ among atoms themselves cannot lead to physically distinct situation, because the identical monoatomic atoms are indistinguishable. Hence, the number of configurations in phase space is divided by $N!$, otherwise too large by factor $N!$. In this way the entropy behaves properly as an extensive quantity, avoiding Gibbs paradox.

When the natural logarithm term decreases below $-5/2$ in Eq. (1), the entropy becomes negative. This is not possible, in contradiction to the third law of thermodynamics. The ST-equation is not valid at low temperature. The concept of a classical ideal gas breaks down at low values of V/N and low values of T . The gas begins to behave as a quantum gas, composed of either bosons or

Corresponding author.

E-mail address: nagatas@almond.ocn.ne.jp (S. Nagata)

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