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A unified approach for the vibration analysis of moderately thick composite laminated cylindrical shells with arbitrary boundary conditions

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ABSTRACT

A unified analytical method based on the first-order shear deformation theory is developed for the vibration analysis of moderately thick composite laminated cylindrical shells subjected to general boundary conditions and arbitrary intermediate ring supports, and various lamination schemes. Each of the displacements and rotation components of the shell, regardless of boundary conditions, is expanded as the linear combination of a standard Fourier series and several supplementary terms are introduced to ensure and accelerate the convergence of the series expansions. Since the displacement field is constructed adequately smooth throughout the entire solution domain, an exact solution can obtained by using Rayleigh–Ritz procedure based on the energy functions of the shell. Furthermore, in contrast to most existing solution procedures, the current method offers a unified solution for laminated cylindrical shells with various boundary conditions, and arbitrary boundary conditions including all classical ones and elastic restraints can be easily achieved by simply setting the stiffnesses of restraining springs without requiring any special procedures or schemes. The excellent accuracy and reliability of current solutions are demonstrated by numerical examples and comparisons with the results available in the literature. The effects of restraining stiffnesses and lamination schemes on frequency parameters are illustrated. Numerous new results for cross-ply and angle-ply laminations with elastically restrained edges and intermediate ring supports are presented, which may serve as benchmark solutions for validating new computational techniques in future.

1. Introduction

Composite materials have found increasing application with the rapid development of industries because they offer advantages over conventional materials. As one of the important structural components, composite laminated cylindrical shells are widely used in various engineering applications, such as naval vehicles aircrafts, and civil industries. The vibration analysis of these cylindrical shells is often required and has always been one important research subject aiming to provide insight into dynamic behaviors and optimal design of complex composite shells.

The increasing use of composite shell structures has motivated great interest in developing more accurate and efficient mathematical model and approaches for analyzing their dynamic behaviors. Significant advances on the dynamic analysis of composite shells have been achieved over the past several decades. A large variety of classical and modern theories and different computational methods have been proposed by researchers, and extensive studies have been carried out based on these theories and methods. The development of researches on this subject has been well documented in several monographs by Leissa [1], Qatu [2], Reddy [3], Carrera [4], Ye [5], and review or survey articles [6–13]. However, a brief review is still necessary in order to properly focus on the features and emphasis of the present paper.

As far as the shell deformation theories reported in previous studies are concerned, there are mainly three major theories which are usually known as: the Classical Shell Theory (CST), the First-order Shear Deformation Theory (FSDT) and the Higher-order Shear Deformation Theory (HSDT). The CST is based on the Kirchhoff–Love assumptions, in which transverse normal and shear deformations are neglected. Various sub-category thin shell theories were developed through different assumptions and simplifications, such as Reissner–Naghdi′s linear shell theory, Donner–Mushtari′s theory, Flügge′s theory, Sanders′ theory and Goldenveizer–Novozhilov′s theory, about which detailed descriptions are available in the monograph by Leissa [1]. Many of the previous studies regarding composite cylindrical shells are based on the CSTs [14–20]. Although sufficiently accurate vibration results for thin

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shells can be achieved using these classical shell theories with appropriate solution procedures, it is inadequate for the vibration analysis of the composite laminated shells which are rather thick or when they are made from materials with a high degree of anisotropic. In such cases, the effects of transverse shear deformations must be considered. Thus, FSDTs based on Reissner–Mindlin′s displacement assumptions were developed to take into account the effects of transverse shear deformations. There exist considerable research efforts devoted to the laminated cylindrical shells based on the FSDTs [12,21–33]. Since the transverse shear strains in the conventional FSDTs are assumed to be constant through the thickness, shear correction factors have to be incorporated to adjust the transverse shear stiffness. Zenkour and Fares [27] presented a refined equivalent single-layer model of anisotropic shells accounting for displacement assumptions of the FSDT and continuous stresses through the shell thickness that are consistent with the surface conditions without the need of any shear correction factors. To overcome the deficiency of the FSDTs and further improve the dynamic analysis of shell structures, a number of HSDTs with varying degree of refinements of the kinematics of deformation were developed [3,6,8,24,28,34–36]. Noticeably, significant contributions to the higher order shear deformation theories of composite shells have been made recently by many researchers [37–43]. As pointed out by Qu et al. [30], although the HSDTs are capable of solving the global dynamic problem of shells more accurately, they introduce rather sophisticated formulations and boundary terms that are not easily applicable or yet understood. And these theories require more computational demanding compared to those FSDTs. Furthermore, from the existing literature, we can know that the first-order theory with proper shear correction factors is adequate for the prediction of the global behaviors of moderately thick laminated shells. Therefore, in the present work, the first-order shear deformation shell theory is just employed to formulate the theoretical model.

Apart from the aforementioned shear deformation theories, it has also been of great interest for researchers to develop an accurate and efficient method which can be used to determine the vibration behaviors of composite laminated cylindrical shells. So far, many computational methods are available for the vibration analysis of cylindrical shells, such as the Ritz method [15,25,44,45], differential quadrature (DQ) method [46–52], Galerkin method [16,53], wave propagation approach [17,18], multiquadric radial basis function method [29,38], meshless method [11], finite element method (FEM) [9,54,55], discrete singular convolution (DSC) approach [56]. Important and excellent review works on the subject have been given by Qatu and co-authors [2,6–8]. Most of these methods were applied firstly to isotropic shells, and were subsequently extended to study the dynamic behaviors of the anisotropic and laminated composite ones.

From the review of the literature, it appears that most of the previous studies on the laminated cylindrical shells are confined to the classical boundary conditions. However, as we know, a variety of possible boundary restraining cases which may not always be classical in nature can be encountered in practice. The existing solution procedures are often only customized for a specific set of different boundary conditions, and thus typically require constant modifications of the trial functions and corresponding solution procedures to adapt to different boundary cases. As a result, the use of the existing solution procedures will result in very tedious calculations and be easily inundated with various boundary conditions because even only considering the classical (homogeneous) cases, one will have a total of hundreds of different combinations. Hence, it is necessary to develop a unified method which is capable of universally dealing with composite cylindrical shells with general boundary conditions. Unfortunately, to the best knowledge of the authors, researches efforts on the topic are very limited. Recently, Qu et al. [20] presented a general domain decomposition method for solving the free, steady-state and transient vibrations of thin composite cylindrical shells subjected to various combinations of classical and non-classical boundary conditions. An improved Fourier series method was previously proposed for the vibration analysis of elastically supported isotropic beams [57] and thin plates [58,59]. Recently, it was extended to the coupled plate structures [60] and thin orthotropic plates [61] with general boundary conditions.

In this present work, an analysis method is developed for the modeling and vibration analysis of moderately thick composite laminated cylindrical shells subjected to general boundary conditions and arbitrary intermediate ring supports, and various lamination schemes, aiming to provide a unified and reasonable accurate alternative to other analytical and numerical techniques. The first-order shear deformation shell theory is adopted to formulate the theoretical model. Each of the displacements and rotation components of the laminated shell, regardless of boundary conditions, is expanded as a standard Fourier cosine series supplemented with auxiliary functions introduced to ensure and accelerate the convergence of the series expansions. Since the displacement field is constructed adequately smooth throughout the entire solution domain, an exact solution is obtained by using Rayleigh–Ritz procedure based on the energy functions of the shell. The accuracy and reliability of current solutions are validated by numerical examples, and the effects of restraining stiffnesses and lamination schemes on frequency parameters are illustrated.

2. Theoretical formulations

2.1. Description of the model

The model considered is shown in Fig. 1(a), the elastically restrained laminated composite circular cylindrical shell is thought to be moderately thick. The length, mean radius and

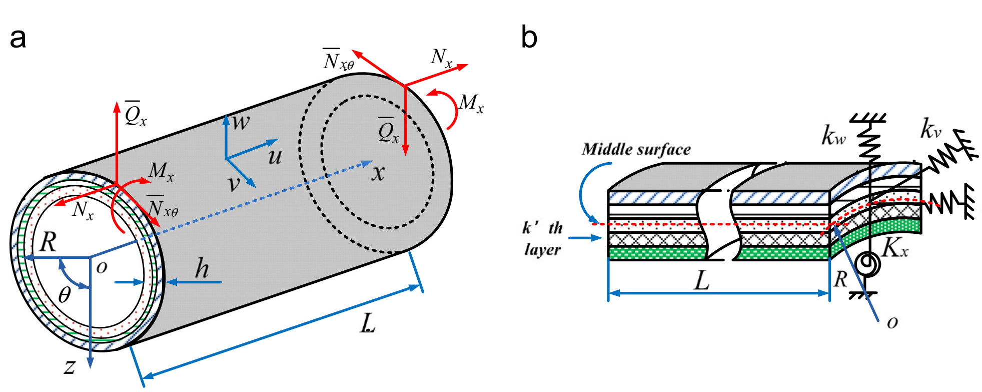

Fig. 1. Schematic diagram of a moderately thick composite laminated cylindrical shell: (a) the whole shell and (b) partial cross-sectional view of the shell.