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Large deflections of shear-deformable cantilever beam subject to a tip follower force

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ABSTRACT

The paper discuses a method for obtaining the equilibrium configurations of a Reissner shear-deformable cantilever beam subject to a tip follower force. Along with the classical follower force, where the angle between the force and normal to the beam cross-section remains constant, the tip rotational load is also discussed. In the latter case there are multiple possible equilibrium configurations of the beam for a given force. The theory is enhanced with numerous numerical examples and examples of deformed beams presented in graphic form.

1. Introduction

The problem of the deflection of a cantilever Euler type elastic beam under tip follower force, which falls into the class of non-conservative problems in elasticity, was first considered as a question of the stability of such a load in the Euler sense [1–3]. In 1980 numerous authors discussed the follower load in the context of development of finite element methods. Among them we mention Argyris and Symeonidis [4] and Alliney and Tralli [5]. It seems that the first discussion of large deflection of the cantilever beam under tip follower force was given by Saje and Srpčič [6]. As a basis they use large deformation beam theory. They consider several load cases and solve the governing two-point boundary value problem with the finite-differences method. The large deflection of a cantilever was considered by Rao and Rao [7,8] as a special case of end rotational load. In the first paper they as the solution of the problem obtain an elliptic integral which gives explicit expression of the load factor as a function of tip angle and the rotation factor. Accordingly, they use the tip angle as a given parameter and then they calculate load parameter. In the second paper they as input data use load parameter and then they calculate tip angle by use the Runge–Kutta integration and the shooting method. The problem of follower load is also discussed briefly by Antman [9], who, with very general constitutive assumptions, considers only the trivial solutions of the problem.

In the recent decade, the problem has attracted new attention. Shkutin [10] consider shear deformable beam and solve the governing sixth-order ordinary differential equations numerically by the shooting method. Zakharov et al. [11] give and analytical solution the problem in terms of Jacobi elliptical functions. Shvartsman [12] considered a large deflection problem for a spring-hinged Euler type beam. Instead of using the shooting method to solve the governing boundary value problem, he, by introducing a new angle variable [9], reduced the problem to an initial value problem which allows direct numerical integration. He then calculates the tip coordinates of a deflected beam by Simpson integration. The group of Indian researchers consider the large deflection of cantilever under tip rotational force [13] and pure follower force [14]. They, by following Rao and Rao [7] analytically express load factor as a function of tip angle and rotational factor and then by numerical integration obtain various shapes of deformed beams. They practically found that—except for the pure follower force—there are multiple possible equilibrium configurations of beam. We note that their method does not allow calculation of the equilibrium configurations for a given force, but only for the given tip angle. Nallathambi et al. [15] numerically solved the problem for a cantilever by changing the direction of integration from beam root to beam tip into integration from tip to root. By this, they transformed the boundary value problem to an initial value problem and in contrast to [12] they for determination of free end slope by using shooting method. Recently Kimiaeifar et al. [16] and Wang et al. [17] solved the problem using the analytical homotopy method.

To that we added that curved cantilever under follower force which was considered by Srpcic and Saje [18], Nallathamb et al. [15]

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and Shvartsman [19]. The later author also consider cantilever beam under two follower forces [20]. We also note that multiple equilibrium configurations for a Euler cantilever beam under dead load was given by Navaee [21] and for sheare-deformable beam preset author [22].

From this review, we can conclude that the problem of determination of equilibrium shapes of a tip follower loaded beam is discussed by most authors only for a Euler type beam. In the present article we extend the analysis of the problem to Reissner shear-deformable beams. First, we formulate the problem and then by the standard method we reduce it to a solution of an ordinary initial value problem. After several simple analytical solutions of the governing equations, we discuss in some detail the possible range of tip angle subject to constitutive restriction and then we add a few numerical examples. We then proceed with a discussion of possible equilibrium configurations of a cantilever beam subject to rotational tip load in some detail. We end the paper with our conclusions.

2. Basic equations

The geometry and load of the beam are shown in Fig. 1. For a description of the deformed state of an initially straight beam we use Reissner equations [23]

$$\frac{dX}{ds}=(1+\varepsilon)\cos\phi-\gamma\sin\phi$$
$$\frac{dY}{ds}=(1+\varepsilon)\sin\phi+\gamma\cos\phi$$
$$\frac{d\phi}{ds}=\kappa \tag{1}$$

In these equations X, Y are coordinates of the deformed beam base curve, ϕ is the tangent angle to the beam, and ε, γ, and κ are successively axial, transverse and bending strain. The parameter s is the length parameter of an undeformed beam measured from beam root to beam tip. For the tip loaded beam the internal force is constant along beam [9]. The axial component N and shear component Q of internal force may be therefore expressed as

$$N=-F\cos(\phi-\phi_1+\alpha)\quad Q=F\sin(\phi-\phi_1+\alpha) \tag{2}$$

where $F>0$ and $\alpha\in[0,\pi]$ are respectively applied force and its direction, and ϕ_1 is the tip angle

$$\phi_1\equiv\phi(1) \tag{3}$$

The moment equilibrium equation $(dM/ds)=\gamma N-(1+\varepsilon)Q$ may be by using expressions for components of internal force (2) written in the form

$$\frac{dM}{ds}=-F[(1+\varepsilon)\sin(\phi-\phi_1+\alpha)+\gamma\cos(\phi-\phi_1+\alpha)] \tag{4}$$

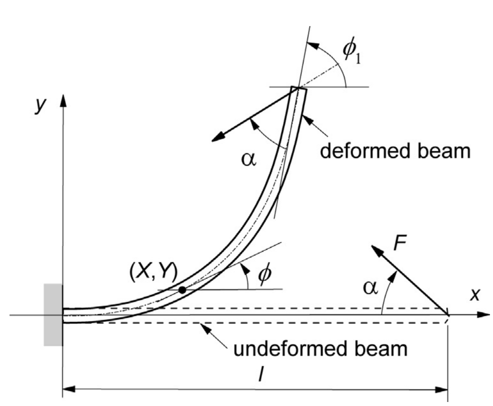

Fig. 1. Geometry and the load of the problem.

We assume that the forces and moment are related to deformations by the following constitutive equations

$$N=EA\varepsilon\quad Q=GA_s\gamma\quad M=EI\kappa \tag{5}$$

where EA, GA_s and EI are positive constants which represent respectively tensile, shear and bending stiffness of the beam.

Boundary conditions for the problem are following

$$X(0)=Y(0)=0 \tag{6}$$

$$\phi(0)=0\quad \kappa(1)=0 \tag{7}$$

Finally, we obtain the length of the deformed beam by integration of the following equation

$$\frac{dS}{ds}=\sqrt{\left(\frac{dX}{ds}\right)^2+\left(\frac{dY}{ds}\right)^2}=\sqrt{(1+\varepsilon)^2+\gamma^2}\quad S(0)=0\quad 0\le s\le L \tag{8}$$

From this, we see that the beam will not stretch to zero at any point for any γ if

$$1+\varepsilon>0 \tag{9}$$

2.1. Non-dimensional form of equations

To reduce the number of parameters in the governing equations we first normalize coordinates to beam length L. Next we introduce the load parameter ω and two non-dimensional material constants: generalized slenderness ratio λ and parameter ν as follows [22]

$$\omega^2\equiv\frac{FL^2}{EI}\quad \frac{1}{\lambda^2}\equiv\frac{EI}{L^2}\left(\frac{1}{EA}+\frac{1}{GA_s}\right)\quad \nu\equiv\frac{GA_s-EA}{GA_s+EA}\in[-1,1] \tag{10}$$

Note that for $GA_s\ll EA$ we have $\nu\to-1$ and the beam becomes shear stiff, while for $GA_s\gg EA$ we have $\nu\to1$ and the beam becomes tensile stiff. We obtain Euler elastica for $1/\lambda^2=0$. Now, by following Antman [9] and Shvartsman [12] we, in order to remove ϕ_1 dependence from the moment Eq. (4), define new angle φ by

$$\varphi\equiv\phi+\alpha-\phi_1 \tag{11}$$

By this we, by using expressions for internal force components (2) and constitutive Eq. (5), obtain the following expressions for axial and shear deformation

$$\varepsilon=-\frac{(1-\nu)\omega^2}{2\lambda^2}\cos\varphi,\ \gamma=\frac{(1+\nu)\omega^2}{2\lambda^2}\sin\varphi \tag{12}$$

This shows that the length of the loaded beam in the case of shear stiff beam shrinks, while a tensile stiff beam always extends. The geometry Eq. (1) can now be by using (10–12) written in the form

$$\frac{dX}{ds}=\cos\phi-\frac{\omega^2}{2\lambda^2}[\cos(\alpha-\phi_1)-\nu\cos(2\varphi+\alpha-\phi_1)]$$
$$\frac{dY}{ds}=\sin\phi+\frac{\omega^2}{2\lambda^2}[\sin(\alpha-\phi_1)-\nu\sin(2\varphi+\alpha-\phi_1)] \tag{13}$$

and (4) may be written in the form

$$\frac{d\varphi}{ds}=\kappa$$
$$\frac{d\kappa}{ds}=-\omega^2\sin\varphi\left[1+\frac{\nu\omega^2}{\lambda^2}\cos\varphi\right] \tag{14}$$

The boundary conditions for (13) remains the same as (6), while the boundary conditions for φ becomes

$$\varphi(0)=\alpha-\phi_1,\ \varphi(1)=\alpha \tag{15}$$