International Journal of Non-Linear Mechanics 50 (2013) 109–126

Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Non-linear dynamic instability of functionally graded plates in thermal environments

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ARTICLE INFO

Article history:
Received 12 April 2012
Received in revised form
17 October 2012
Accepted 17 October 2012
Available online 26 October 2012

Keywords:
Non-linear vibrations
FGM plates
higher-order shear-deformation
in-plane load
buckling
parametric excitation

ABSTRACT

Geometrically non-linear parametric instability of functionally graded rectangular plates in thermal environments is investigated via a multi-degree-of-freedom energy approach. Non-linear higher-order shear deformation theory is used and the non-linear response to in-plane static and harmonic excitation in the frequency neighborhood of twice the fundamental mode is investigated. The boundary conditions are assumed to be simply supported movable. Numerical analyses are conducted by means of pseudo arc-length continuation and collocation technique to obtain force–amplitude relations in the presence of temperature variation in the thickness direction. The effect of volume fraction exponent and temperature variation on the onset of instability for both static and periodic in-plane excitation are fully discussed and the post-critical non-linear responses are obtained. Moreover, direct time integration of equations of motion is carried out and bifurcation diagrams, phase-space plots, Poincaré maps and time histories are obtained showing complex non-linear dynamics through period-doubling and Neimark–Sacker bifurcations.

1. Introduction

Functionally graded (FG) plates have received considerable research efforts, recently. These structures are inhomogeneous composites fabricated from a mixture of metal and ceramic with smooth and continuous variation of material properties through the thickness. In comparison to isotropic and laminated plates they have reduced thermal stresses and stress concentrations and have the capability of withstanding high temperature gradient environments without losing structural integrity. These advantages together with high strength and light weight have made functionally graded materials a suitable replacement for conventional materials in aeronautical vehicles as thermal barriers. Therefore, studying the vibration characteristics of FG structures under large amplitude thermo-mechanical loads is of paramount importance.

Extensive surveys on the topic of non-linear vibrations of plates can be found in the books of Chia [1] and Amabili [2]. Amabili [3–5] has studied the non-linear vibrations of rectangular plates subjected to transverse harmonic excitation. Based on a Lagrangian approach and using an arc-length continuation code, the non-linear behavior of plates with different boundary conditions were given in [3], comparison to experimental results for plates with geometric imperfections and non-conventional boundary conditions were presented in [4], and the thermal effects on non-linear vibrations were studied in [5].

When subjected to in-plane loads, plates and shells may lose stability and buckle after a certain threshold. In fact, when static in-plane loads are applied, the instability occurs through pitchfork bifurcation while in case of periodic in-plane loads the so-called parametric instability occurs through period-doubling bifurcation in the frequency neighborhood of twice the fundamental frequency [2]. In the latter case and in contrast with primary resonance due to transverse excitation, the instability may arise even for small excitation amplitudes and much below the static buckling load. This phenomenon has been greatly studied for cylindrical shells by Pellicano and Amabili [6] and Pellicano [7].

Early investigations on the dynamic stability of plates under in-plane pulsating load are those of Bolotin [8], Somerset and Evan-Iwanowski [9] and Yamaki and Nagai [10]. In these studies, Galerkin technique was utilized to reduce the governing equations to a system of coupled ordinary differential equations of Mathieu-Hill type and the Bolotin's [8] technique was used to obtain the instability regions. The application of finite element method and finite difference method in studying the dynamic stability of plates was shown by Hutt and Salma [11] and Singh and Dey [12], respectively. Nguyen and Ostiguy [13,14] studied the effect of different boundary conditions on the dynamic

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0020-7462/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijnonlinmec.2012.10.009

instability and non-linear response of isotropic plates both theoretically and experimentally. Awrejcewicz and Krysko [15] and Awrejcewicz et al. [16], showed that thin rectangular plates are susceptible to chaotic response through Hopf bifurcation followed by a Feigenbaum scenario when subjected to in-plane periodic loads. A comprehensive review of developments in the parametric instability of plates can be found in Evan-Iwanowski [17] and Sahu and Datta [18].

A complete survey on the topic of non-linear analyses of FG plates can be found in the book by Shen [19]. Studies on the post-buckling [20–27] and non-linear free vibration analysis [28–30] of FG rectangular plates are quite abundant. However, dynamic stability analyses of FG plates subjected to periodic in-plane loads can be hardly found in the literature [31–33]. The first contribution on the dynamic stability of FG plates was made by Ng et al. [31] who used a combination of discretized Lagrangian approach and Bolotin's technique to study the parametric resonance of simply supported FG plates under periodic in-plane load at fixed temperature. Wu et al. [32] studied the dynamic stability of FG simply supported and clamped plates using moving least squares differential quadrature technique. Yang et al. [33] used higher-order shear deformation theory with a combination of differential quadrature method and Bolotin's technique to obtain dynamic stability region of thick FG plates. It should be noted that in Refs. [31–33], non-linear effects were neglected and therefore only dynamic stability boundaries were obtained and the prediction of the post-bifurcation state remained unsolved.

Studies on the chaotic vibrations of FG plates are scarce [34–36]. Among them, Alijani et al. [34] studied the chaotic vibrations of FG rectangular plates in thermal environment when subjected to transverse harmonic load. The equations of motion were obtained by using the first-order shear deformation theory and, in order to classify the system dynamics, bifurcation diagrams were obtained using Gear's backward differentiation formula and Lyapunov exponents were calculated. Hao et al. [35] and Zhang et al. [36] studied the chaotic response of FG plates under simultaneous in-plane and transverse harmonic loads with the same excitation frequency and at fixed temperature using asymptotic perturbation technique and method of multiple scales, respectively. In both studies, higher-order shear deformation theory was used. However the effect of in-plane and rotary inertia were neglected and the discretized governing equations of motion had simply two [35] and three [36] degrees of freedom.

Moreover, in studying non-linear behavior of FG plates, the effective material properties are evaluated either by using a simple rule of mixture (Voigt model) [20–23,26–37] or by considering the Mori–Tanaka micromechanics scheme [24,25,29], which is usually applicable to regions of the graded microstructure with well-defined continuous matrix and discontinuous particulate phase [19]. Recently, it has been shown by Shen and Wang [37] that the difference between the fundamental frequencies of FG plates obtained via Voigt model and Mori–Tanaka are very small, and in case of non-linear frequencies (non-dimensionalized with respect to the corresponding natural frequencies) are negligible. In addition to the different micromechanics models that are usually considered, the material properties of the constituent materials are either assumed to be temperature independent (e.g. [23,24]) or temperature dependent (e.g. [31–37]). Since functionally graded structures are mostly used at high temperature environments, their mechanical properties should be considered temperature dependent. In most of the studies with temperature dependent material properties only the modulus of elasticity and the coefficient of thermal expansion were assumed to be temperature dependent while the thermal conductivity coefficient was assumed to be temperature independent (e.g. [19–21,31–37]), which cannot be very accurate. Furthermore, it is well explained in Refs. [19,22,28,30] that considering temperature dependent material properties has an almost negligible effect on the non-linear static and dynamic behavior of FG structures.

In the present paper, geometrically non-linear parametric vibrations of FG plates in thermal environments are studied using a multi-degree-of-freedom Lagrangian approach and non-linear higher-order shear deformation theory. To model the material properties of the FG plate, the Voigt model is used and the material properties are assumed to be temperature independent. The boundary conditions of the plate are simply supported movable and the temperature is assumed to vary only in the thickness direction. The effect of in-plane loads is included in the virtual work done by the external forces. The energy functional is reduced to a system of infinite non-linear ordinary differential equations with quadratic and cubic non-linear term. A bifurcation analysis is carried out by means of pseudo arc-length continuation and collocation scheme. In particular, at first the solution starts from the trivial undistributed configuration of the plate at zero thermal or static in-plane force and is incremented to a desired temperature or static force level. The result of the first stage is the post-buckled state of the plate which is the initial condition for the dynamic part. Then, in order to obtain the dynamic force–amplitude response, the bifurcation analysis is continued from the post-buckled state by considering the amplitude of the harmonic force as the bifurcation parameter. The effect of temperature variations, volume fraction exponents and thickness ratio are deeply discussed. It is shown that, when FG plates are subjected to in-plane static loads or temperature changes in thickness direction, the pitchfork bifurcation (representing buckling) that is obtained for isotropic plates is destroyed and a continuous buckling behavior is obtained.

Moreover, it is shown that similar to isotropic plates, FG plates lose stability through period-doubling bifurcation when subjected to in-plane periodic loads. It is revealed that temperature variations expedite the onset of instability and that the post-critical state obtained after the period-doubling bifurcation is complex and is followed by a jump and a Neimark–Sacker bifurcation. Furthermore, in order to have a better perspective of the underlying dynamics, direct time integration of the equations of motion is performed using Gear's backward differentiation formula to obtain bifurcation diagrams, Poincaré maps and time histories which indicate sub-harmonic, quasi-periodic and chaotic motions.

2. Equations of motion

2.1. Basic equations

A rectangular plate with in-plane dimensions a and b and thickness h is considered in an orthogonal coordinate system $(O;\ x,\ y,\ z)$, as shown in Fig. 1. The plate is made of functionally graded materials (FGMs) with continuous variation of constituents from metal rich surface at the bottom ($z=-h/2$) to ceramic rich surface at the top ($z=h/2$) and it is assumed that the plate is subjected to pulsating compressive in-plane force $\tilde{N}(t)$ in y direction. Since in this paper the focus is on non-linear dynamics of FG plates and not on stress distribution or linear behavior, the temperature dependency of material properties and micromechanical modeling become of less importance, and thus the material properties can be written according to a power law distribution as follows:

$$E(z)=E_m+(E_c-E_m)\left(\frac{2z+h}{2h}\right)^k,\quad \alpha(z)=\alpha_m+(\alpha_c-\alpha_m)\left(\frac{2z+h}{2h}\right)^k, \tag{1a, b}$$

$$\kappa(z)=\kappa_m+(\kappa_c-\kappa_m)\left(\frac{2z+h}{2h}\right)^k,\quad \rho(z)=\rho_m+(\rho_c-\rho_m)\left(\frac{2z+h}{2h}\right)^k, \tag{1c, d}$$