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Secondary torsional moment deformation effect in inelastic nonuniform torsion of bars of doubly symmetric cross section by BEM

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ABSTRACT

In this paper a boundary element method is developed for the inelastic nonuniform torsional problem of simply or multiply connected prismatic bars of arbitrarily shaped doubly symmetric cross section, taking into account the secondary torsional moment deformation effect. The bar is subjected to arbitrarily distributed or concentrated torsional loading along its length, while its edges are subjected to the most general torsional boundary conditions. A displacement based formulation is developed and inelastic redistribution is modeled through a distributed plasticity model exploiting three dimensional material constitutive laws and numerical integration over the cross sections. An incremental–iterative solution strategy is adopted to resolve the elastic and plastic part of stress resultants along with an efficient iterative process to integrate the inelastic rate equations. The one dimensional primary angle of twist per unit length, a two dimensional secondary warping function and a scalar torsional shear correction factor are employed to account for the secondary torsional moment deformation effect. The latter is computed employing an energy approach under elastic conditions. Three boundary value problems with respect to (i) the primary warping function, (ii) the secondary warping one and (iii) the total angle of twist coupled with its primary part per unit length are formulated and numerically solved employing the boundary element method. Domain discretization is required only for the third problem, while shear locking is avoided through the developed numerical technique. Numerical results are worked out to illustrate the method, demonstrate its efficiency and wherever possible its accuracy.

1. Introduction

In engineering practice we often come across the analysis of members of structures subjected to twisting moments. Curved bridges, ribbed plates subjected to eccentric loading or columns laid out irregularly in the interior of a plate due to functional requirements are most common examples. Moreover, design of bars and bar assemblages based on elastic analysis are most likely to be extremely conservative not only due to significant difference between initial yield and full plastification in a cross section, but also due to the unaccounted for yet significant reserves of strength that are not mobilized in redundant members until after inelastic redistribution takes place. Thus, material nonlinearity is important for investigating the ultimate strength of a bar that resists torsional loading, while distributed plasticity models are acknowledged in the literature [1–3] to capture more rigorously material nonlinearities than cross sectional stress resultant approaches [4] or lumped plasticity idealizations [5,6].

When an elastic bar is subjected to uniform torque arising from two concentrated torsional moments at its ends while the warping of the cross section is not restrained, the angle of twist per unit length remains constant along the bar. Under these conditions, the bar is leaded to uniform torsion and the well known primary (St. Venant) shear stress distribution arises forming the primary torsional moment stress resultant [7]. When arbitrary torsional boundary conditions are applied either at the edges or at any other interior point of a bar due to construction requirements, this bar under the action of general twisting loading is leaded to nonuniform torsion and additional normal and secondary (warping) shear stresses arise [8], forming the warping moment and secondary torsional moment stress resultants, respectively. In order to include warping shear stresses in the global equilibrium of the bar, that is to account for the secondary torsional moment deformation effect (STMDE), an additional kinematical component (along with the angle of twist) is generally required (see for example [9,10]), increasing the difficulty of the problem at hand.

The STMDE has been shown in the literature to be significant, especially on closed shaped section bars. Massonnet [11] presents

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a qualitative explanation why warping shear stresses are of the same order of magnitude as primary ones in the case of closed shaped section bars. As early as 1954, Benscoter [12] analyzed the nonuniform torsional problem of multicell section bars. Since then, a significant amount of relevant contributions has appeared in the literature as well [10,13–17]. Since the topic at hand is analogous to the geometrically nonlinear Timoshenko beam theory of shear-bending loading conditions [18,19], it does not satisfy local equilibrium equations (for relevant discussions see for example Simo et al. [20] and Minghini et al. [21]). This problem is alleviated by introducing torsional shear correction factor at the global level [19,22,23] and suitable warping shear stress distribution at the local level [24,25,22]. The aforementioned contributions refer to the linear elastic regime. If inelastic effects are considered, especially through distributed plasticity formulations, then the nonuniform [26,27] torsion problem including STMDE requires a much more rigorous analysis.

Apart from research efforts in which bars are idealized with computationally demanding three dimensional [28] or shell [29] elements, several researchers proposed specialized beam elements to analyze bars under inelastic nonuniform torsion [1,26,27,30–39]. In some of these contributions the inclusion of warping shear stresses in the global equilibrium of the bar has been achieved. For example, Wunderlich et al. [35] employed a power series numerical technique using an Updated Lagrangian formulation to study thin-walled beams under general loading conditions. Nie and Zhong [36] and Wang et al. [37] studied thin-walled beams under general loading conditions by employing an Updated Lagrangian description and the FEM. Alsafadie et al. [38] formulated a mixed corotational beam element to study thin-walled beams under general loading conditions. Gruttmann et al. [39] analyzed beams of arbitrarily shaped cross section under general loading conditions employing the FEM. In these publications, two 1-D kinematical components are employed to model nonuniform torsion. However torsional shear correction factor is not included in the analyses, while the employed warping shear stress distributions do not satisfy local equilibrium considerations under inelastic or even elastic conditions as well.

A different approach is undertaken in the recent contribution of Sapountzakis and Tsipiras [40] where a single (fourth-order) governing differential equation is formulated with respect to a single kinematical component (angle of twist), leading to an approximate solution of the problem at hand. However, the adopted warping shear stress distribution verifies local equilibrium equations under elastic conditions. Moreover, an alternative methodology is presented in the very recent contribution of Le Corvec and Filippou [41], where a multitude of local section warping degrees of freedom are introduced in order to model effects such as in-plane plasticity effects, constrained warping, shear lag, etc. This technique does not require the introduction of torsional shear correction factor, however the reduction of the number of employed degrees of freedom requires further investigation, while an example of a thick-walled cross section beam is not presented. In the publication of Wackerfuß and Gruttmann [42], beams of thick-walled rectangular cross sections are worked out by employing a series of polynomials as global warping functions in order to capture advanced effects such as in-plane inelastic redistribution and transverse contraction. Moreover, the same researchers [43] have employed local warping functions to capture the aforementioned effects in moderately thick arbitrarily shaped cross section beams, however bimoment loading cannot be applied at the bar. Finally, it is worth here noting that the BEM has not yet been employed to the inelastic nonuniform torsional problem of bars, with the exception of the aforementioned work of Sapountzakis and Tsipiras [40].

In this paper a boundary element method is developed for the inelastic nonuniform torsional problem of simply or multiply connected prismatic bars of arbitrarily shaped doubly symmetric cross section taking into account the effect of secondary torsional moment deformation. The bar is subjected to arbitrarily distributed or concentrated torsional loading along its length, while its edges are subjected to the most general torsional boundary conditions. A displacement based formulation is developed and inelastic redistribution is modeled through a distributed plasticity model exploiting three dimensional material constitutive laws and numerical integration over the cross sections. An incremental–iterative solution strategy is adopted to resolve the elastic and plastic part of stress resultants along with an efficient iterative process to integrate the inelastic rate equations [44]. The one dimensional primary angle of twist per unit length, a two dimensional secondary warping function and a scalar torsional shear correction factor are employed to account for STMDE. The latter is computed employing an energy approach under elastic conditions [22]. Three boundary value problems with respect to (i) the primary warping function, (ii) the secondary warping one and (iii) the total angle of twist coupled with its primary part per unit length are formulated and numerically solved employing the boundary element method [45]. Domain discretization is required only for the third problem, while shear locking is avoided through the developed numerical technique. The essential features and novel aspects of the present formulation compared with previous ones are summarized as follows:

(i) For the first time in the literature, STMDE is taken into account to the problem at hand and its influence is quantified. Evaluation of both St. Venant and warping shear stresses is based on the solution of boundary value problems formulated by exploiting local equilibrium considerations under elastic conditions. A torsional shear correction factor is also employed to capture STMDE more rigorously. It is computed using an energy approach under elastic conditions.
(ii) The formulation is a displacement based one taking into account inelastic redistribution along the bar axis by exploiting three dimensional material constitutive laws and numerical integration over the cross sections (distributed plasticity approach). The plastic part of the secondary twisting moment stress resultant is successfully taken into account through the proposed approach.
(iii) The cross section is an arbitrarily shaped thin- or thick-walled doubly symmetric one. The formulation does not stand on the assumptions of a thin-walled structure and therefore the cross section's primary torsional, secondary torsional and warping rigidities are evaluated "exactly" in a numerical sense.
(iv) An incremental–iterative solution strategy is adopted to resolve the elastic and plastic part of stress resultants. Integration of the inelastic rate equations is performed for each monitoring station with an efficient iterative process and the plastic part of stress resultants is obtained employing incremental strains.
(v) The developed procedure retains most of the advantages of a BEM solution over a pure domain discretization method, although it requires domain discretization to the longitudinal problem, exhibiting the following features:
  - Shear locking is avoided by employing the same order of approximation for both the total and the primary part of the angle of twist per unit length.
  - Cross sectional discretization is employed exclusively for numerical integration of domain integrals.
  - Finite differences and differentiation of shape functions are not required.