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A numerical investigation of developing flow in an eccentric curved annulus in the presence of gravity

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In this article developing incompressible viscous flow in an eccentric curved annulus in the presence of gravity is numerically studied using a second order finite difference method based on the projection algorithm to solve the governing equations including the continuity and full Navier–Stokes equations. The equations written in a bipolar–toroidal coordinate system are discretized in a three dimensional staggered grid. The effects of governing non-dimensional parameters including the eccentricity, non-dimensional curvature ratio, Dean number, Froude number, aspect ratio, and the Reynolds number on the flow field in the entrance and fully developed region are investigated. The numerical results indicate that at the small Froude numbers, the flow field distorts from the symmetrical condition due to the larger body force effect and the axial velocity formation mostly takes place at the lower half of the annulus. In addition, at the constant Froude number, by decreasing the curvature radius, the peak axial velocity and its sharp gradient appear on the outer curvature region due to the larger centrifugal forces and by increasing the eccentricity the flow rate intensifies at the wider region and weakens at the narrower region due to the larger flow resistance. Furthermore, the friction factor increases by decreasing the Froude number and increasing the Dean number.

1. Introduction

Curved pipes have many applications in different science fields such as bio-fluid mechanics especially the blood flow in the catheterized artery, piping systems, cooling systems of rotating electrical machinery, lubrication systems, aerospace industries, chemical mixing or drying machinery, chemical reactors, chromatography columns, and other processing equipment. Because of the wide range of applications, flow in these configurations is studied extensively during the last decades.

Physical aspects of the fluid flow inside the curved pipes are very much complicated due to the presence of curvature which generates the centrifugal and pressure forces in the curvature direction. In contrast to the centrifugal forces, the pressure forces decrease in the curvature direction as the fluid particles approach the centre of curvature. Mutual effects of centrifugal, pressure, inertia, viscous and gravitational forces provide a very complex flow pattern, which has not physically fully understood. Yao and Berger [1] have carried out a relatively detailed qualitative physical description of the flow in a plain curved pipe. However, shifting from a plain curved pipe flow to the eccentric curved annulus makes the flow pattern more complex owing to the presence of an additional internal curved pipe as well as the eccentricity effect. In this case, the secondary boundary layers, which are formed due to the secondary flow [2], develop on the walls of both curved pipes. At the absence of gravitational force, these boundary layers start from their outermost point of curvatures, where the pressure forces are more than the centrifugal forces. On the other hand, in the core region, off the two pipe walls, the reverse fluid motion, i.e., from the inner to the outer radii of curvature, occurs resulting from the larger centrifugal force. This secondary core flow which starts from the symmetrical plane at the inner radius of curvature ($\varphi=\pi$) develops similar to a jet flow and interacts with the opposite flow of secondary boundary layers, forming vortices. The presence of gravitational force results in the displacement of secondary boundary layers and vortices, which is the focus of this paper.

This phenomenon implies a physical point that in secondary flows the diffusion of viscous forces occurs more rapidly than the main axial flow owing to the presence of small inertia forces (order of magnitude of secondary inertia forces is about 10^{-1} of the axial one). The presence of secondary flows in the curved pipes delays the development of the main flow by deforming the evolution of the axial velocity profile and increases the resistant of the fluid flow. Consequently, the increase of both friction rate and entrance length in the curved pipe, comparing to the similar straight one, results in the flow rate reduction.

Studies on the fluid flow inside an eccentric annulus have been carried out for straight annuli considering either horizontal or vertical cases, which include both the fully developed and

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Nomenclature

a	radius of curvature and the pole of the bipolar coordinate system
D	diameter
D_h	hydraulic diameter
e	eccentricity
f	average friction factor
f_i	local friction factor on inner wall
f_o	local friction factor on outer wall
f_{cs}	concentric straight annulus friction factor
Fr	Froude number
FR	normalized friction factor
h	coordinate scale factor
h_η	coordinate scale factor in the η direction
h_ξ	coordinate scale factor in the ξ direction
h_φ	coordinate scale factor in the φ direction
N	radius ratio
p	pressure
r	radius
Re	Reynolds number
t	time
u,v,w	velocity components in η, ξ, φ direction
V	velocity vector
W_m	mean axial velocity

Greek symbols

δ	curvature ratio
ε	dimensionless eccentricity
η	first bipolar coordinate
μ	viscosity of fluid
κ	dean number
ξ	second bipolar coordinate
ρ	density of fluid
φ	curvature coordinate

Subscripts

i	inner wall
m	mean value
o	outer wall

developing flows. Cheng and Hwang [3], Trombetta [4], and Susuki et al. [5] have studied eccentric annuli using the analytical and numerical tools. The study by Manglik and Fang [6] has investigated the effect of eccentricity in the laminar fully developed conditions.

Feldman et al. [7] have investigated developing flow inside eccentric annuli using a finite difference method. They have shown that the hydrodynamic entrance length increases as the eccentricity increases. A numerical study in concentric and eccentric horizontal cylindrical annuli has been carried out by Ho et al. [8]. Hirose et al. [9] have conducted a numerical analysis and experiments on the eccentric horizontal annuli at different orientations. They have found that the heat transfer rate varies by the eccentricity and oriented angle. Furthermore, an experimental and numerical study have been performed by Naylor et al. [10] to explore the eccentricity effects on the heat transfer rate at different range of Rayleigh numbers. The study by Sathyamurthy et al. [11] has focused on a vertical eccentric annulus where a finite volume approach has been employed to solve the governing equations. On the other hand, Choudhury and Karki [12] have investigated a horizontal eccentric annulus using a numerical approach and they determined the eccentricity effect on the heat transfer rate and friction factor. In addition, other numerical works on the flow and heat transfer inside vertical annuli have been performed by several authors [13–16]. All of the pervious works published involve flow in the eccentric straight annuli.

Since the aim of this study is developing flow in eccentric curved annuli, it is necessary to review some of the works done on the curved pipe flows. The first major study on the flow in a curved pipe was made by Dean [17,18]. He considered a loosely curved pipe in which the flow depends on a single non-dimensional parameter, i.e., the Dean number., $K=2a/R(w_{\max}\ a/\upsilon)^2$, where a is the radius of pipe, R is the radius of curvature, $w_{\max}$ is the maximum axial velocity in the corresponding straight pipe, and υ is the kinematic viscosity. Dean's work is valid for $K \leq 576$. In later works on the curved pipes, a variety of Dean numbers have been used by different researchers. For example McConalogue and Srivastava [19] proposed the parameter $D=(Ga^2/\mu)(2a^3/\upsilon^2 R)^{1/2}$, where G is the constant pressure gradient along the pipe. This parameter relates to K as $D=4\,K^{1/2}$. They considered intermediate range of Dean numbers ($96 \leq D \leq 600$) using the Fourier series method to formulate the problem and to solve the resulting equations numerically. Collins and Dennis [20] and Dennis [21] used a finite difference method to solve the flow equations in the range of $96 \leq D \leq 5000$. An investigation on the developing laminar flow in a curved pipe was made by Soh and Berger [22] using the artificial compressibility technique. They found that the curvature ratio has a large effect on the intensity of secondary flow and on the separation, which occurs near the inner wall of the curved pipe. Among other similar works on the flow in a curved pipe, the works of Pedley [23], Dennis and Ng [24], Ito [25] and Kao [26] can be mentioned.

Nobari and Gharali [27] have investigated the effect of internal fins on the fluid flow and heat transfer through a rotating straight pipe and a stationary curved pipe. Ishigaki [28–30] examined the flow in a rotating curved pipe and investigated the effect of Coriolis force on the flow structure. Flow in a curved annular pipe has been studied by Karahalios [31] and Petrakis and Karahalios [32] in the fully developed region. The studies by Karahalios [33], Ebadian [34], Jayaraman and Tiwari [35], and Dash et al. [36] have considered the effect of catheterization on the flow characteristics in a curved artery. Recently Nobari et al. [37] have investigated developing flow in a curved concentric annular pipe. Later on Nobari and Mehrabani [38] have studied flow and heat transfer in the eccentric curved annuli in the fully developed region.

It has to be noticed that secondary flow can be generated in the straight ducts by other physical effects rather than the curvature in the curved ducts. For instance, Speziale has theoretically proven that [39] turbulent secondary flow develops in the straight pipes with non-circular cross section due to the non-zero difference in the normal Reynolds stresses on planes perpendicular to the axial flow direction. In a similar work, Huang and Rajagopal [40] have studied the necessary and sufficient conditions for the fully developed turbulent secondary flow of a Newtonian fluid in a straight tube under the influence of a conservative body force. They also have shown that in the absence of a body force, a sufficient condition exists for turbulent secondary flow in terms of the transverse normal stress differences associated with the Reynolds stress. The study by Mollica and Rajagopal [41] has shown that secondary flows can develop