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The non-linear dynamics of FGM truncated conical shells surrounded by an elastic medium

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ABSTRACT

In this study, the non-linear dynamic analysis of functionally graded (FG) truncated conical shells surrounded by an elastic medium has been investigated using the large deformation theory with von Karman–Donnell-type of kinematic non-linearity. The material properties of FG truncated conical shell are assumed to vary continuously through the thickness direction. The Pasternak model is used to describe the reaction of the elastic foundation on the FG conical shell. The fundamental relations, the non-linear motion and compatibility equations of FG truncated conical shells surrounded by an elastic medium are derived. By using the Superposition method, Galerkin method and Harmonic balance method, the problem of non-linear vibration of the FG truncated conical shell surrounded by an elastic medium is solved. Finally, the influences of variations of the elastic medium, compositional profiles and conical shell characteristics on the frequency–amplitude relations are investigated. The present results are compared with the available data for a special case.

1. Introduction

The idea of the construction of functionally graded materials (FGMs) was first introduced in 1984 by a group of Japanese materials scientists [1]. The principal developments in the area of FGMs, research progress and their various possible applications are summarized in the literature [2–5]. Due to high performance of heat resistance capacity and excellent characteristics in comparison with conventional composites, FGMs which are microscopically composites and composed from mixture of metal and ceramic constituents have attracted considerable attention recent years. By continuously and gradually varying the volume fraction of constituent materials through a specific direction, FGMs are capable of withstanding ultrahigh temperature environments and extremely large thermal gradients. Therefore, these novel materials are chosen to use in structure components of aircraft, aerospace vehicles, nuclear plants, machines as well as various temperature shielding structures widely used in industries. This intelligent challenge motivated many research centers in various countries to launch intensive and extensive research projects with the aim of investigating the dynamic behavior of FGM shell structures. Extensive investigations are done on the non-linear vibration of FGM shell structures and mainly focused on the cylindrical shells [6–14].

A complete survey on this topic can be found in a book by Shen [9]. Recently, the static and dynamic analyses of FGM conical shells have been studied, but to a far lesser extent than the FGM cylindrical shells [15–23].

On the other hands, FGM cylindrical and conical shells on elastic foundations have been widely adopted by many researchers to model interaction between elastic media and these structural elements for various engineering problems. Sheng and Wang [24] studied thermal vibration, buckling and dynamic stability of functionally graded cylindrical shells embedded in an elastic medium. Shen [25] and Shen et al. [26] presented the postbuckling response of a shear deformable functionally graded cylindrical shell of finite length embedded in a large outer elastic medium and subjected to axial compressive loads and internal pressure in thermal environments. Shah et al. [27] studied vibrations of functionally graded cylindrical shells based on elastic foundations. Bagherizadeh et al. [28] studied mechanical buckling behavior of functionally graded material cylindrical shells surrounded by Pasternak elastic foundation. The vibration and buckling analyses of FGM truncated conical shells under different loads and resting on the two-parameter elastic foundation were recently studied by Sofiyev and his co-authors [29–31].

It should be noted that all of these works are limited to the linear vibration or stability of FGM cylindrical and conical shells surrounded by an elastic medium, and the model of elastic medium is taken into account as the Pasternak-type. The non-linear vibration behaviors for FGM cylindrical and conical shells resting on the Pasternak-type elastic foundation are very few in

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the open literature. After an extensive literature review, only one article is found, in which the non-linear vibration of shear deformable FGM cylindrical shells surrounded by elastic medium is examined [32].

The non-linear vibration characteristics of FG truncated conical shells on elastic foundations of great interest to design and manufacture, in connection with the wide use in many fields, such as a missile, aviation and submarine technology, machines etc. Mathematically, such problems are quite complicated, in both formulation and solution (use of the theory of elasticity and the theory of shells, formulation of medium–shell interface conditions, development of solution methods, etc.). In the present work, attention has been focused on the non-linear free vibration of of FGM truncated conical shells resting on the Pasternak-type elastic foundation. The fundamental relations, the non-linear motion and compatibility equations of the FGM truncated conical shell surrounded by an elastic medium are derived. By using the Superposition method, Galerkin method and Harmonic balance method, the problem of non-linear vibration of FGM truncated conical shells surrounded by an elastic medium is solved. Finally, the influences of variations of the elastic medium, compositional profiles and conical shell characteristics on the frequency–amplitude relations are investigated.

2. Problem formulation

2.1. Effective material properties of FGMs

As shown in Fig. 1 a thin FGM truncated conical shell is considered. The structure is referred to a curvilinear coordinate system (S,θ,z), where S and θ axes lie along the generator and in the circumferential direction on the reference surface of the cone, respectively and the z axis, being perpendicular to the plane of the first two axes, lies in the inwards normal direction of the cone. γ denotes the semi-vertex angle of the cone, L is the length and h is the thickness of the FGM truncated conical shell. R_1 and R_2 indicate the radii of the cone at its small and large ends, respectively. S_1 and S_2 are the distances from the vertex to the small and large bases, respectively. Also, u, v and w denote displacement (due to loads) of a point in the reference surface in the direction of a generator, the circumferential direction, and the inward normal direction, respectively.

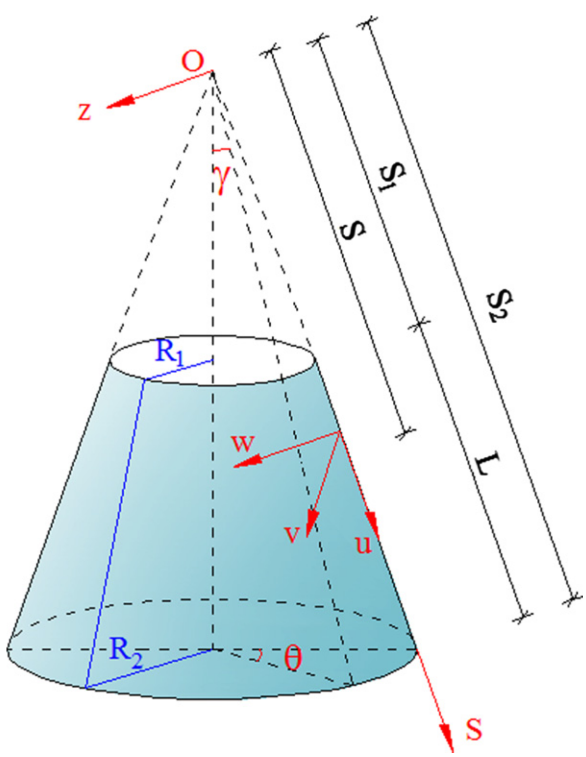

Fig. 1. Geometry of the truncated conical shell.

The truncated conical shell material exhibits perfectly elastic behavior. Mechanical properties of the material are distributed over the shell thickness according to power law. We confine our attention to functionally graded materials fabricated by mixing two material phases with different properties. Usually metal and ceramic phases are mixed in such a manner that their volume fractions denoted by V_m and V_c, respectively, grade in the thickness direction of the shell. It is common practice to approximate continuous volume fraction distribution across the truncated conical shell thickness by the power law [20]:

$$1.\text{Linear}: V_c = \bar{z} + 0.5, \quad \bar{z} = z/h \tag{1.1}$$

$$2.\text{Quadratic}: V_c = (\bar{z} + 0.5)^2 \tag{1.2}$$

$$3.\text{InverseQuadratic}: V_c = 1 - (0.5 - \bar{z})^2 \tag{1.3}$$

$$4.\text{Cubic}: \quad V_c = 3(\bar{z} + 0.5)^2 - 2(\bar{z} + 0.5)^3 \tag{1.4}$$

The ceramic and metal volume fractions are related by

$$V_c + V_m = 1 \tag{2}$$

To approximate variation of any material, a property denoted by P as show in Eq. (3) can be used. It is reasonable to assume that the same simple rule of mixture holds:

$$P = P_c V_c + P_m V_m \tag{3}$$

where P_c and P_m are the material properties of the ceramic and metal, respectively [33].

From Eqs. (2) and (3), the effective Young's modulus, Poisson's ratio and density of FGMs can be written as

$$E(\bar{z}) = E_{cm} V_c + E_m, \quad \nu(\bar{z}) = \nu_{cm} V_c + \nu_m, \quad \rho(\bar{z}) = \rho_{cm} V_c + \rho_m \tag{4}$$

where $E_{cm} = E_c - E_m$, $\nu_{cm} = \nu_c - \nu_m$, $\rho_{cm} = \rho_c - \rho_m$, and E_m, ν_m, ρ_m and E_c, ν_c, ρ_c are the Young's modulus, Poisson's ratio and density of the metal and ceramic surfaces of the FGM truncated conical shell, respectively.

2.2. Analytical model of elastic medium

The FGM truncated conical shell is surrounded by an elastic medium (Fig. 2). Pasternak model is used to describe the reaction of the elastic medium on the conical shell. If the effects of damping and inertia force in the foundation are neglected, the foundation interface pressure $N(S,\varphi)$ may be expressed as

$$N(S,\varphi) = K_w w - K_p \left(\frac{\partial^2 w}{\partial S^2} + \frac{1}{S}\frac{\partial w}{\partial S} + \frac{1}{S^2}\frac{\partial^2 w}{\partial \varphi^2} \right) \tag{5}$$

where $\varphi = \theta \sin\gamma$, K_w(in N/m^3) is the Winkler foundation stiffness and K_p(in N/m) is the shear subgrade modulus of the elastic foundation [29,34].

The stiffness is characterized by (K_w, K_p) for the Pasternak-type elastic foundation model, by (K_w, 0) for the Winkler elastic foundation model, and by (K_w,K_p) = (0,0) for the foundationless model.

3. Basic equations

According to von Karman non-linear strain–displacement relations, the strain components on the middle plane of truncated