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Analytical modeling of the interface crack between a piezoelectric actuator and an elastic substrate considering shear effects

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ABSTRACT

In this paper, the behavior of an interface crack between a piezoelectric actuator and an elastic substrate considering shear effect is studied. Based on the Timoshenko beam theory, analytical solutions are obtained for calculating interface stresses and mode I and II energy release rates of a straight crack in piezoelectric composite adhesive interface subjected to mechanical–electrical loadings, and the adhesive layer is modeled as a continuous spring with the shear and peel stiffness. The energy release rates and stress intensity factors predicted by the present analytical solutions agree well with those available in the literatures. And the influences of the applied electrical loading, geometry and the material mismatch upon the characterization of interface crack onset for the typical piezoelectric composite structure are discussed. The present analytical solutions include the effects of shear and interface deformations. The analytical model and conclusions provided in this paper would contribute to better understanding interface failure of piezoelectric smart structures, and benefit the interface design and interface safety assessment of piezoelectric composite structures.

1. Introduction

Due to their intrinsic coupled mechanical–electrical behavior, piezoelectric materials are of great importance in aerospace, navigation, civil, mechanical engineering, etc. However, interface fracture in adhesively bonded smart structures presents an important concern in multilayer devices, especially under coupled mechanical–electrical loading, during which interface crack propagation is often observed [1]. Moreover, to enable the implementation of piezoelectric actuators in various structural members without significantly complicating the manufacturing process or changing the mechanical properties of the target structure, piezoelectric sheets, patches and tiles are usually adhesively bonded to the outer faces of the substrate in many practical cases, introducing an additional adhesive layer into the structural assembly. Adhesive interface fracture of piezoelectric composite structure is, therefore, of paramount importance and has drawn much attention [2–4].

Some researchers have addressed the experimental study of interface fracture of piezoelectric composite structures. Du et al. [5] developed an experimental technique which involved using piezoelectric actuators to apply cyclic loading to study crack growth along polymer–aluminum interface. Shang et al. [6] studied the interface delamination behavior of multilayered Cr/PZT/PLT/Pt/Ti thin films deposited on single crystal Si substrate, a series of sandwiched-cantilever type specimens were tested, but the experimental results only provided the fracture loads of these specimens, which cannot be used to characterize the interface toughness. Woo et al. [7] examined the fracture process of a thin monolithic PZT and a plate-type piezoelectric composite actuator(PCA) subjected to a three-point bending loading, the results showed that the damage was initiated in the brittle PZT layer, which induced the delamination between PZT layer and adjacent fiber composite layers which lead to the final failure of PCAs. However, specialized methodologies for quantifying adhesive fracture toughness for piezoelectric structures are limited due to unique experimental challenges, as piezoelectric ceramic materials are brittle and weak tensile fracture strength, and also extremely difficult to machine into test geometries specified by ASTM, JIS, and ISO test standards. Hence, the fracture criterion for an interface crack in piezoelectric composite materials is still unclear. To this point, only reasonable experimental data can tell the real situation. Recently, Bai et al.[8] developed an experimental system, based on fixed-ratio mixed-mode (FRMM) loading and digital image correlation (DIC) method, to study crack growth along piezoelectric-composite adhesive interface, the deformation and strain fields were obtained by DIC method, and the beam theory was used to calculate the critical fracture energies for piezoelectric composite FRMM specimens.

For a piezoelectric composite structure, debonding or interface fracture between two adjacent layers is a typical failure mechanism. The early attempt to analysis interface crack problem of

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piezoelectric material was to model a finite crack at the interface of a piezoelectric bimaterial subjected to a far-field uniform tension by fracture mechanics method [9]. To significantly simplify the calculations, the problem of an interface crack between dissimilar anisotropic piezoelectric materials was analyzed by introducing a complex variable approach presented in later research [10]. Then, Narita et al. [11] and Kwon et al. [12]solved the anti-plane (mode III) problem of an interface crack between a piezoelectric material and an elastic material. Ru [13] obtained the exact solution for two bonded dissimilar piezoelectric half-planes with discontinuous thin electrode layers embedded at the interface. Then, significant attention has also been paid to the modeling of debonding of actuators. Liu et al. [14] studied the interface crack between a piezoelectric actuator and an elastic substrate under in-plane electric loading. By means of Fourier transformation methods, the governing equations are converted to an integral equation, which is then converted to a standard Hilbert problem. A closed form solution for stresses, electric field, and electric displacements along the bonded interface is obtained. Wang et al. [15] examined the coupled electromechanical behavior of a thin piezoceramic actuator embedded in or bonded to an elastic medium under inplane mechanical and electrical loadings. The actuator is characterized by an electroelastic line model with the poling direction being perpendicular to its length, but the model did not consider the adhesive layer and thus may not be used to investigate the peel effects. Jin et al. [16] further modified the electroelastic line model presented by Wang et al. [15] by considering the effect of the mechanical and geometrical properties of the adhesive layer on the coupled mechanical–mechanical behavior of a thin piezoelectric ceramic actuator bonded to an elastic medium, and the effects of interface debonding on the response of the layered structure and on the interlaminar strain and stress transfer mechanisms were discussed. However, among the above mentioned work, most of them either are limited to the infinite thickness of the piezoelectric biomaterial or elastic substrate or have no explicit expressions for fracture parameters. In most practical engineering applications, however, the dimension of the layered piezoelectric structures is finite, demanding a general model from which results for the finite layers can be derived.

Further studies have been carried out to enhance the developed analytical solutions by considering various geometrical and material as well as load combinations. An analytical model that focuses on the strain and stress transfer mechanisms in piezoelectrically actived panels has been presented by Crawley et al. [17]. In this analysis, the axial stress in the actuator was assumed to be uniform across its thickness and the substrate was treated as a Euler–Bernoulli beam. The result indicated that, for a perfectly bonded actuator, the shear stress between the actuator and the host beam was transferred mainly at the ends of the actuator. Crawley et al. [18] later developed a Euler–Bernoulli model of a piezoelectric actuator by considering the linear stress distribution along its thickness. Im and Atluri [19] further modified the actuator model presented by Crawley et al. [17] by considering both the axial and the transverse shear forces in the beam. Tong et al. [20,21] based on the classic theory in bonding joints, a theoretical model for a PZT smart beam including adhesive layers was developed, in which PZT patches and the host beam were modeled as Euler–Bernoulli beams, and the adhesive layer was modeled as a continuous spring with the shear and peel stiffness, and the edge debonding was also studied by simply considering a shortened PZT length, and analyzed effects of debonding on the distributions of strain, stress and displacement. Although numerous researchers have well established the Euler–Bernoulli beam model, modeling of the smart structures by shear deformable theory is limited. Shear effects in the distributed control of a laminated beam become important when a high degree of accuracy is crucial and for beams with length to thickness ratio less than 15 for isotropic materials and less than 30 for composite materials. Since most, if not all smart structures contain composite layer, aspect ratio of 30 or less is quite possible and hence shear effects cannot be ignored. Consequently, the Timoshenko beam can be essential for providing reliable results [22]. Recently, Qiao et al. [23] proposed an interface deformable piezoelectric bi-layer beam model to study the electromechanical responses and the interface stress distributions in a smart layered structure. The layer-wise approximations of both the elastic displacements and electric potential are employed, and each sub-layer is modeled as a single linearly elastic Timoshenko beam perfectly bonded together through a deformable interface by introducing two interface compliance coefficients. However, this model does not really consider the geometrical and material properties of the adhesive interface, and the two interface compliance coefficients must be determined through another semi-analytical and semi-numerical calibrating process, such as in the elastic foundation model.

Based on the Timoshenko beam theory, we develop an analytical model to study the mechanical–electrical behavior of a piezoelectric actuator bonded to an elastic substrate with an interface crack, and simple and closed-form solutions of interface stresses and mode I and II energy release rates are obtained. An adhesive layer between the piezoelectric actuator and the elastic substrate is introduced which is modeled as a continuous spring with the shear and peel stiffness. The energy release rates are applicable to a general geometrical and material combination of piezoelectric composites and are expressed in terms of electrical and mechanical force at the cross section of the crack tip. The present solutions are then compared with results available in the literatures. Good agreement between analytical and literatures solutions is achieved. And the influences of the applied electrical loading, geometry and the material mismatch upon the characterization of interface crack onset are discussed in detailed. Finally, conclusions are provided to manifest the merits as well as limitations of the present model and shed light on the directions for further study.

2. Interface crack model including adhesive layer

2.1. Basic models and assumptions

Let us consider two semi-infinite length media, as a piezoelectric actuator bonded to a homogeneous and isotropic elastic substrate on the top surface through an adhesive layer, as illustrated in Fig. 1. In this model, the piezoelectric actuator and elastic substrate are modeled as Timoshenko beams, and the adhesive layer is modeled as a continuous spring with the shear and peel stiffness. It is assumed that a straight crack propagate along midline of the adhesive layer that is sandwiched between piezoelectric actuator and elastic substrate of different thickness. The poling direction of the piezoelectric actuator is along the negative z-axis. The free body diagrams for the infinitesimal elements of the piezoelectric actuator, adhesive layer and elastic substrate are depicted in Fig. 2.

2.2. Fundamental formulation

By referring to Fig. 2, the equilibrium equations for piezoelectric actuator and elastic substrate can be written as:

$$\begin{cases} \dfrac{dN_1}{dx}=-\tau, \dfrac{dN_2}{dx}=\tau \\ \dfrac{dQ_1}{dx}=-\sigma, \dfrac{dQ_2}{dx}=\sigma \\ \dfrac{dM_1}{dx}=Q_1-\dfrac{h_1}{2}\tau, \dfrac{dM_2}{dx}=Q_2-\dfrac{h_2}{2}\tau \end{cases} \tag{1}$$