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Analysis of circular free edge effect in composite laminates by *p*-convergent global–local model

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ABSTRACT

In this paper the *p*-convergent global–local model based on layerwise theory is presented to predict the complicated patterns of stress fields around a circular hole of composite laminates under tension. A distinction of this model is to combine two-dimensional elements with three-dimensional elements in a designed mesh. In the local region with high stress gradient, three-dimensional displacement fields can be defined by layer-by-layer representation, while equivalent single-layer elements are adopted in the global region with smooth stress gradient. Also, the *p*-refinement in local as well as global regions is simultaneously implemented using Lobatto shape functions. Higher-order shape functions for three-dimensional elements are derived by the combination of one- and two-dimensional shape functions in a layerwise sense. In this study the orders of the shape functions are kept to be fixed as *p*-level (in-plane direction)=8 and *q*-level (thickness direction)=5 from the convergence test of in-plane and transverse stresses. The proposed model achieves compatibility displacements and stress equilibrium at the junction or interface between the different element types. Also, exact mapping of curved boundary is undertaken using blending functions. Numerical examples of curved free-edge problems have been taken into account to illustrate the performance of the present approach. Numerical results show that the proposed model is capable of predicting in-plane stresses around a circular hole as well as interlaminar stresses at the interface between layers.

1. Introduction

Increasing application of composite laminates is evident in a variety of engineering structures and manufactured components. However, some problems have often been encountered for engineering application. Among the problems, free-edge effects have aroused much attention as the high localized stresses coming from the stiffness discontinuity between layers occur in the vicinity of free edges. Especially, for many cases of engineering application, the interlaminar stresses are very important for prediction of delamination which is one of the critical failure types of laminated systems. It is also well known that the presence of interlaminar stress components is particularly important in the regions close to free and loaded edges like bolted joints. Because of the presence of singularity of the stress fields close to edges and in correspondence of interfaces between layers, the prediction of the stresses close to the edge of laminated structures is a very complicated problem to be solved. In the case of curved free edges, it would be more difficult to obtain rational results than in straight free edges, whatever theoretical or numerical approach is used.

A variety of methods have been proposed for the interlaminar stresses at straight free edges of laminated plates under in-plane uniaxial loading. The work on the stresses that occurred at straight free edges can be found in the review articles [1,2]. However, laminated composite plates with curved free edges received relatively less attention. Only a limited number of authors have investigated the interlaminar stresses around holes in the laminated composite plates under in-plane loading. Tang [3] developed the boundary-layer theory only to satisfy the part boundary conditions in an average sense. Analytical approaches may encounter difficulties to accurately predict the free-edge stresses. Rybicki and Schmueser [4] adopted a three-dimensional element modeling for interlaminar stresses in the vicinity of circular holes. Ericson et al. [5] described the distribution of interlaminar stresses using three-dimensional singular elements. To reduce the calculating work of computers, Zhang and Yeng [6] proposed a simplified approach suitable for only limited layer orientations. For interlaminar stresses around circular hole in symmetric laminated plates, Ko and Lin [7] used an analytical approach. Barboni et al. [8] employed six-node triangular elements based on the multilayer higher-order theory. Hu et al. [9] predicted the distribution of interlaminar stresses around curved free edges by three-dimensional finite elements with much large computational effort. Thus, Raghuram and Murthy [10] attempted to develop the two-dimensional theory of coupled bending and extension for the straight

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and curved free-edge problems. Ukadgaonker and Rao [11] proposed the general solutions for free-edge stresses of laminated composites with arbitrary shapes of holes. Pan et al. [12] used three-dimensional boundary elements for the free-edge stress problems. Recently, some researchers [13,14] developed single-layer higher-order models with mesh refinements for the curved free-edge problems subjected to extensional, bending as well as thermal loads.

Meanwhile, although several attempts have been made to develop simple and efficient elements using displacement models satisfying only C°-continuity requirements, these elements sometimes failed to predict some stresses accurately when applied to plates with free edges. The reason for the failure of these displacement finite elements can be attributed to the use of a lower-order displacement field that is not adequate for predicting the variation of stresses which defined by higher-order derivatives of the displacements. Thus, the quest for robust finite elements has triggered researchers to develop higher-order finite elements, so-called framework of the p-version of the finite element method. The mathematical justification showing the advantages of using a higher-order approximation of the field variables including high accuracy, high convergence rate with coarser meshes, and improved performance in handling stress singularity problems was given by Babuska et al. [15].

As compared to use conventional three-dimensional analysis for laminated composite plates with curved free edges, simpler analysis to obtain rational results is more attractive. In this study, the global–local model based on the p-version finite element method is proposed for interlaminar stresses at the curved free edges in laminated composite plates. The analysis is implemented by dividing the entire domain into the global and the local regions. In the local region with high stress gradient, three-dimensional displacement fields can be captured by layer-by-layer representation, while equivalent single-layer elements are adopted in the global region with smooth stress gradient. Also, the p-refinement in local and global regions is simultaneously implemented on the basis of Lobatto shape functions [16] being hierarchical and orthogonal. Also, for the shape functions to approximate three-dimensional displacement fields, the higher-order shape functions are derived by combination of the one- and two-dimensional shape functions, not using pure higher-order three-dimensional shape functions. The compatibility between global and local regions is enforced by reducing a continuum element through the kinematic constraints compatible with shell deformations. Using the laminated models based on the p-version finite element concept, Ahn et al. [17] and Ahn and Basu [18] implemented fracture analysis for patch repaired plates. In this paper, the p-convergent global–local model adopts the blending function method [19] to describe curved boundaries exactly. Thus the simplicity of the proposed modeling scheme allowing a very coarse mesh is shown. Also, some results obtained by the present model are verified with several test problems reported in some literatures.

2. *p*-Convergent global–local model based on Lobatto shape functions

2.1. Approximation of displacement fields

This approach with quadrilateral elements is based on the concept of subparametric elements. Thus approximate functions of displacement and geometry fields are separated. Firstly, in any layer k, displacement fields of the proposed p-convergent global–local model can be written as

$$\begin{aligned} U^k(x,y,z) &= u_L^k(x,y,z)+\overline{u}_L^k+u_G(x,y,z)+\overline{u}_G \\ V^k(x,y,z) &= v_L^k(x,y,z)+\overline{v}_L^k+v_G(x,y,z)+\overline{v}_G \\ W^k(x,y,z) &= w_L^k(x,y,z)+\overline{w}_L^k+w_G(x,y,z)+\overline{w}_G \end{aligned} \tag{1}$$

where u_L^k,v_L^k, and w_L^k are local displacements with respect to nodal modes, and u_G,v_G and w_G are global displacements. Also, bars placed over symbols refer to nodeless modes contrary to nodal modes. Because the variables corresponding to nodeless modes do not have coordinates, one only considers nodal modes in modeling of laminated plates and then nodeless modes are created by coordinates of the variables of nodal modes. The local components with respect to nodal modes can be written as:

$$\begin{aligned} u_L^k(x,y,z) &= N_iF_ju_i^j \\ v_L^k(x,y,z) &= N_iF_jv_i^j, \quad i=1,2,3,4; \quad j=1,2 \\ w_L^k(x,y,z) &= N_iF_jw_i^j \end{aligned} \tag{2}$$

where F and N are the one- and two-dimensional shape functions, respectively, based on Lagrange interpolation functions. The additional local components with respect to nodeless modes are given by

$$\begin{aligned} \overline{u}_L^k &= N_iB_sa_i^s+M_tF_jb_k^j+M_tB_sc_k^s && i=1,2,3,4; j=1,2; \\ \overline{v}_L^k &= N_iB_sd_i^s+M_tF_je_k^j+M_tB_sf_k^s && t=1,2,\ldots,\tfrac{p(p+3)}{2}-1; \\ \overline{w}_L^k &= N_iB_sg_i^s+M_tF_jh_k^j+M_tB_sm_k^s && s=1,2,\ldots,q-1 \end{aligned} \tag{3}$$

where the symbols, a, b, c, d, e, f, g, h and m indicate nodeless variables. B and M are one- and two-dimensional Lobatto shape functions [16], respectively. Also, q (thickness direction) and p (in-plane direction) refer to orders of one- and two-dimensional approximate functions used, respectively. The global displacement components with respect to nodal variables can be expressed as follows:

$$\begin{aligned} u_G(x,y,z) &= N_i\left(u_i+z\theta_i^x\right) \\ v_G(x,y,z) &= N_i\left(v_i+z\theta_i^y\right), \quad i=1,2,3,4 \\ w_G(x,y,z) &= N_iw_i \end{aligned} \tag{4}$$

Lastly, the global displacement components with respect to nodeless variables are given by

$$\begin{aligned} \overline{u}_G &= M_i\left(a_i+z\varphi_i^x\right) \\ \overline{v}_G &= M_i\left(b_i+z\varphi_i^y\right), \quad i=1,2,\ldots,\frac{p(p+3)}{2}-1 \\ \overline{w}_G &= M_ic_i \end{aligned} \tag{5}$$

The hierarchical characteristics and computational robustness of these shape functions are well established [20]. For a typical layer k, a stress–strain relationship in a local region is based on the three-dimensional elasticity theory,

$$\left\{\sigma_{x,y,z}\right\}_{6\times1}^k = [D]_{6\times6}^k\left\{\varepsilon_{x,y,z}\right\}_{6\times1}^k \tag{6}$$

Here, $[D]$ is a general elasticity matrix of a orthotropic material and a strain matrix is given by

$$\left\{\varepsilon_{x,y,z}\right\} = \left\lfloor \frac{\partial U}{\partial x}\frac{\partial V}{\partial y}\frac{\partial W}{\partial z}\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\frac{\partial U}{\partial z}+\frac{\partial W}{\partial x}\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right\rfloor^{\mathrm{T}} \tag{7}$$

Also, in a global region, the constitutive relationship of elements with respect to x, y, and z axes can be defined as:

$$\left\{\sigma_{x,y,z}\right\}_{8\times1} = [L]_{8\times8}\left\{\varepsilon_{x,y,z}\right\}_{8\times1} \tag{8}$$

where $[L]_{8\times8}$ refers to a full elasticity matrix with n layers and is composed of sub-matrices as shown below:

$$[L]_{8\times8} = \sum_{i=1,2,..}^{n}\begin{bmatrix} [E]_{3\times3}^i & [C]_{3\times3}^i & [0]_{2\times2} \\ [C]_{3\times3}^i & [S]_{3\times3}^i & [0]_{2\times2} \\ [0]_{2\times2} & [0]_{2\times2} & [Q]_{2\times2}^i \end{bmatrix}_{8\times8} \tag{9}$$

$[E]$ is the extensional elasticity matrix, $[S]$ is the bending elasticity matrix, $[C]$ is the coupling between bending and extensional elasticity matrices, $[Q]$ is the transverse-shear elasticity matrix, and $[0]$ denotes a null matrix. The summation accounts