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Free vibration analysis of thin and thick-walled FGM box beams

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ABSTRACT

Free vibration of FGM box beam is investigated by the formulation of an exact dynamic stiffness matrix on the basis of first-order shear deformation theory (FSDT). Primary and secondary torsional warping, shear and bending deformations are incorporated in the one-dimensional beam model. Material properties of the beam are assumed to be graded across the wall-thickness. A recent function characterising the distribution of shear stresses is introduced. The present model is validated by comparison with finite element analysis for various boundary conditions. Good correlation exists between the values of Abaqus analysis and those calculated with the present method.

1. Introduction

Composite box beams have found increasing applications in a variety of engineering fields such as aerospace, civil, and marine during the recent decades. It becomes an obvious trend that more and more this type of beams will be used in the design of structural components such as aircraft wings, helicopter rotor blades, robot arms, bridge decks and other structural elements in civil constructions. However, the use of the composite materials was limited by the high temperature until the appearance of new material known as functionally graded material (FGM). Due to its inherent smooth and continuous variation of material properties along some preferred direction, many scientists are attracted by the FGM. Recently, El Meiche, et al. [1] presented buckling and vibrations analysis of simply supported rectangular FGM sandwich plate, a new hyperbolic shear deformation theory was introduced in their model in order to derive the governing equations by applying Hamilton's principle. Hosseini-Hashemi, et al. [2] developed exact closed-form solutions for dynamic analysis of rectangular FGM plates with different boundary conditions on the basis of the FSDT, they investigated the influence of the volume fraction and Geometry on the free vibration characteristics of FGM plates. Pradhan and Murmu [3] used modified differential quadrature method (MDQM) based on Euler–Bernoulli beam theory to solve governing differential equations of FGM sandwich beam under variable elastic foundations, this study was carried out with various temperature distributions, volume fractions, variable Winkler foundation modulus and normalized core thickness of FGM beams. Ke et al. [4] and Kitipornchai et al. [5] focused their attention to the study of postbuckling response and nonlinear vibration of hinged–hinged clamped–hinged clamped–clamped FGM beams containing an open edge crack, the Ritz method was employed to derive their nonlinear governing equations. Aydogdu and Taskin [6] analysed free vibrations of simply supported FGM beams with comparative study between the classical beam theory (CBT), higher-order shear deformation theory (HSDT) [7,8] and FSDT. It is observed from the literature that there are many interesting approaches to analyse the behavior of classical anisotropic box beams. Kim and White [9,10], McCarthy and Chattopadhyay [11] developed three-dimensional composite thin and thick-walled box beams theories under static loads, the governing equations for three types of specially tailored layups with specialised elastic couplings known as the cross-ply layup configuration, the circumferentially asymmetric stiffness (CAS) configuration, and the circumferentially uniform stiffness (CUS) configuration are considered in their works, finally, the results have been validated with experimental data [12], analytical predictions [13] and refined beam FEA [14]. Subsequently, Volovoi et al. [15] evaluated most composite beams theories. Pluzsik and Kollàr [16] developed a theory for orthotropic box beams subjected to a static sinusoidal load and compared it with that of Vlasov [17] which is more efficient for open section isotropic beams effects of restrained warping and shear deformation were investigated with numerical results. Vo and Lee [18] developed a finite element model to solve the problem of flexural–torsional coupled vibration of thin-walled

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composite box beams with arbitrary lay-ups under a constant axial force, the effects of axial force, fiber orientation and modulus ratio on the natural frequencies, load-frequency interaction curves and corresponding vibration mode shapes were investigated with numerical examples. However, the literature on the analysis of the FGM box beams is very few. Librescu et al. [19] studied the behavior of FGM thin-walled beams at high temperatures, which included instability and vibration analysis along with the effects of temperature gradients and volume fraction. Piovan and Machado [20] adopted a second-order non-linear displacement field in order to study the dynamic stability of simply supported thin-walled box beams made of FGM under an axial external force, the effects of shear deformation, volume fraction index and the interaction between forced and parametrically excited vibrations on the boundaries of the unstable regions have been investigated but the shear effects across the wall-thickness was neglected, therefore, the primary purpose of the current work is to fill this gap for studying the dynamic behavior of FGM box beam, the primary and secondary torsional warping function obtained by the author [21] is introduced in displacement fields, a recent function satisfying the shear-stress-free boundary conditions at top, bottom, left, and right of the box beam [1] is used to correct shear stresses. A simple Newton's eigenvalue iteration method is adopted in the present analysis to determine all the natural frequencies.

2. Kinematics

Consider a symmetric FGM box beam of length L, minimum cross-sectional dimension c, maximum cross-sectional dimension d and wall thickness h (see Fig. 1). The Cartesian coordinate system (x, y, z) and the curvilinear system (x, s, n) are used. The coordinate s is measured along the tangent to the middle surface in a counter-clockwise direction, while n is the coordinate perpendicular to the s coordinate. The origin of the coordinates is set at the center of beam cross-section. The properties are graded along the wall-thickness h and depend only on the variable n.

The power law is given by

$$E(n) = (E_t - E_b)[(n/h) + (1/2)]^p + E_b \tag{1a}$$

$$\rho(n) = (\rho_t - \rho_b)[(n/h) + (1/2)]^p + \rho_b \tag{1b}$$

where E_t and E_b denote values of the elasticity modulus while ρ_t and ρ_b are mass density at $n = h/2$ and $n = -h/2$, respectively and p is a variable parameter which dictates the material variation profile through the thickness.

To develop the present model, a number of assumptions are stipulated:

(a) The beam cross-sections are assumed rigid in their own planes.
(b) Transverse shear stresses vary parabolically across the minimum and maximum cross-sectional dimensions.
(c) Torsional primary and secondary warping are included in this formulation.
(d) This model is developed in the context of small deformations within linear elasticity.
(e) The Poisson's coefficient ν is assumed to be constant.

In general, the displacements, u, v and w of any generic point on the profile section in the x, y and z directions, respectively, may be expressed as

$$u(x,y,z,t) = u_0(x,t) - y\phi_y(x,t) - z\phi_z(x,t) - [\psi_p(y,z) + \psi_s(y,z)]\theta'(x,t) \tag{2a}$$

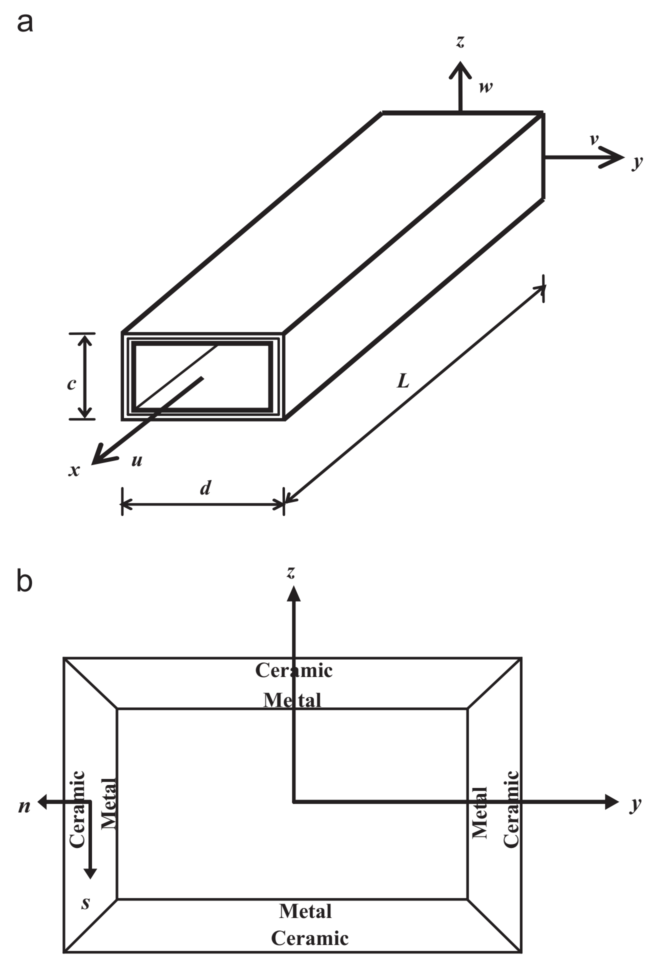

Fig. 1. Geometry and material variation of the FGM box beam.

$$v(x,z,t) = v_0(x,t) - z\theta(x,t) \tag{2b}$$

$$w(x,y,t) = w_0(x,t) + y\theta(x,t) \tag{2c}$$

where u_0, v_0 and w_0 are the mid-plane displacements in the x, y, and z directions, while the variables ϕ_y, ϕ_z and θ denote the rotations about the z, y and x axes, respectively. The superscript primes denote the partial derivatives with respect to x.

The primary and secondary torsional warping functions ψ_p and ψ_s are replaced by that of [21] in Eq. (2d).

$$\begin{aligned}\psi(y,z) &= \psi_p(y,z) + \psi_s(y,z) \\ &= -yz + \frac{8d^2}{\pi^3} \\ &\quad \times \sum_{i=0}^{\infty} \frac{\sin(((2i+1)\pi y)/d)\sinh(((2i+1)\pi z)/d)\sin(((2i+1)\pi)/2)}{(2i+1)^3\cosh(((2i+1)\pi c)/2d)}\end{aligned} \tag{2d}$$

The strains associated with the displacements in Eq. (2) are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = u_0' - y\phi_y' - z\phi_z' - \psi\theta'' \tag{3a}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = v_0' - \phi_y - \left(z - \frac{\partial \psi}{\partial y}\right)\theta' \tag{3b}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = w_0' - \phi_z + \left(y - \frac{\partial \psi}{\partial z}\right)\theta' \tag{3c}$$