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Review

Turbulent field helicity fluctuations and mean helicity appearance

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ABSTRACT

There are many experiments where nonzero mean turbulent helicity is measured either directly or indirectly. Despite the study of mechanisms of its occurrence, the question of mean helicity generation remains open. In our paper, we explore the emergence of the mean helicity in the turbulent field created by an external random force with zero helicity under a simultaneous external large-scale impact (rotation or homogeneous magnetic field). It is shown that anisotropy (even weak one) results in the appearance of mean helicity and in the emergence of an analog of the diamagnetic effect in the vortex field.

Contents

1. Introduction

The notion of helicity as one of principal features of turbulent flows was introduced about half a century ago. However, its role in the turbulence behavior and, in particular, its influence on turbulence energy redistribution remain far from being clear until now. At present, helical turbulence with nonzero mean helicity $H_t = \langle \boldsymbol{u} rot \boldsymbol{u} \rangle$ (where $\boldsymbol{u}$ is a fluctuating component of the velocity field, and $\langle ... \rangle$ is averaging over a volume or an ensemble) is taken into account when solving many problems connected with generation processes in turbulent media. At that, side by side with turbulent energy flux $\varepsilon = dE/dt$ over the scales, helicity flux $\eta = dH_t/dt$ is also introduced, which is the main key parameter for the description of spectral behavior of turbulence [1].

Nevertheless, the appearance of nonzero mean helicity in various media remains an open question. As a rule, it is assumed that nonzero helicity primordially exists (which is, surely, possible) and intensifies under an external impact [2–14]. In the present paper, we examine large-scale mechanisms leading to the appearance of mean helicity He_t in turbulent flows, where initially $He_t = 0$.

A two-point velocity correlator of a homogeneous and isotropic incompressible turbulent flow with violated mirror symmetry can be represented in a general form (see, e.g., [1]):

$$\langle u_i(\mathbf{r},t) u_j(0,t) \rangle = A(r)\delta_{ij} + B(r) r_i r_j + C(r)\varepsilon_{ijk} r_k \tag{1}$$

where $u_i(\mathbf{r},t)$ is a fluctuating component of the velocity field in the point $\mathbf{r}$ at the moment t, $A(r)$, $B(r)$ and $C(r)$ are functions depending on the modulus of $\mathbf{r}$, ε_{ijk} is a completely antisymmetric unit tensor.

Fourier-presentation of this correlator (1) in a general form is as follows:

$$\langle u^*{}_i(k,t) u_j(k,t) \rangle = \frac{E(k,t)}{4\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + i\varepsilon_{ikj} \frac{H(k,t)}{8\pi k^4} k_k \tag{2}$$

where $E(k,t)$ and $H(k,t)$ are energy and helicity densities, respectively, and k is a wave vector. On the other hand, as we know, mean helicity $H_t = C(0)$ is a measure of mirror symmetry disturbance in a turbulent flow. However, if $H_t = 0$, "latent asymmetry" of the flow could have effect even in the mirror symmetry

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turbulence [15,16]. The same is applicable, as shown below to the external force with a zero gyrotropic part, but with a "latent gyrotropy".

If the mirror symmetry of the flow is not violated, $H_t=0$. However, it does not mean that helical vortices do not form in the flow. Just as previously, we can expand the velocity and vortex fields in helical waves $h_s(\mathbf{k})\exp(i\mathbf{kr})$ (see, for example, [17,18]), i.e.

$$u(\mathbf{r},t)=\sum_{\mathbf{k}}\sum_{s}a_s(\mathbf{k},t)h_s(\mathbf{k})\exp(i\mathbf{kr})$$

and

$$\omega(\mathbf{r},t)=rot(u(\mathbf{r},t))=\sum_{\mathbf{k}}\sum_{s}s|\mathbf{k}|a_s(\mathbf{k},t)h_s(\mathbf{k})\exp(i\mathbf{kr}).$$

In this case, spectral densities of energy and helicity can be represented as a sum and difference of their positively defined helical components $E^{\pm}(k,t)$ and $H^{\pm}(k,t)$, which are defined as follows:

$$E^{\pm}(k,t)=\frac{1}{2}\sum_{\mathbf{p}}\langle|a_{\pm}(\mathbf{p},t)|^2\rangle\delta(k-|p|)$$

and

$$H^{\pm}(k,t)=\sum_{\mathbf{p}}|\mathbf{p}|\langle|a_{\pm}(\mathbf{p},t)|^2\rangle\delta(k-|p|)$$

and are interconnected as follows: $H^{\pm}(k,t)=2kE^{\pm}(k,t)$, whereas

$$E(k,t)=E^{+}(k,t)+E^{-}(k,t);\ H(k,t)=H^{+}(k,t)-H^{-}(k,t) \tag{3}$$

and $\langle...\rangle$ denotes averaging over an ensemble.

If $H(k,t)=0$ and $H^{+}(k,t)=H^{-}(k,t)$, then $E^{+}(k,t)=E^{-}(k,t)=1/2E(k,t)$.

Balance equations for spectral densities of energy and helicity lead to the following equations for $E^{\pm}(k,t)$ and $H^{\pm}(k,t)$ [18]:

$$\frac{\partial E^{\pm}}{\partial t}=T_E^{\pm}(k,t)-2\nu k^2E^{\pm}+F_E^{\pm};\quad \frac{\partial H^{\pm}}{\partial t}=T_H^{\pm}(k,t)-2\nu k^2H^{\pm}+F_H^{\pm} \tag{4}$$

where $T_X^{\pm}(k,t)$ is a flux of the respective component over the spectrum, and $F_X^{\pm}(k,t)$ is a function of its source.

$H(k,t)\neq 0$ only when $F_H(k,t)=F_H^{+}(k,t)-F_H^{-}(k,t)\neq 0$, which leads to the asymmetry of Eq. (4) and to the appearance, as in (2), of a gyrotropic term in the expression of the force correlator density in the Fourier-space. However, in this case H_t can equal zero.

Then,

$$\begin{aligned}\langle u_i^*(k,t)u_j(k,t)\rangle^{+}&=\frac{E^{+}(k,t)}{4\pi k^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)+i\varepsilon_{ikj}\frac{H^{+}(k,t)}{8\pi k^4}k_k\\ \langle u_i^*(k,t)u_j(k,t)\rangle^{-}&=\frac{E^{-}(k,t)}{4\pi k^2}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)-i\varepsilon_{ikj}\frac{H^{-}(k,t)}{8\pi k^4}k_k\end{aligned} \tag{5}$$

whose sum gives Eq. (2), while their difference is:

$$\begin{aligned}&\langle u_i^*(k,t)u_j(k,t)\rangle^{+}-\langle u_i^*(k,t)u_j(k,t)\rangle^{-}\\ &\quad=\frac{H(k,t)}{8\pi k^3}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)+i\varepsilon_{ikj}\frac{E(k,t)}{4\pi k^3}k_k\end{aligned} \tag{6}$$

i.e., helicity is a measure of energy difference in heteropolarized helical modes.

As $2k(E^{+}(k,t)-E^{-}(k,t))=H(k)$ is a non-compensated part of helical modes with the energy density (see also Eq. (6)).

$$E_H(k,t)=|E^{+}(k,t)-E^{-}(k,t)|=\left|\frac{H(k,t)}{2k}\right|, \tag{7}$$

it can be considered as a separate substance existing in a compensated medium (see also [18]).

Thus, the total helicity takes part in the energy transfer as an additional channel, and for each k, energy emission or absorption by components of a symmetric field occurs (because $H(k)$ is an alternating quantity). Meanwhile, helicity growth in certain scales (see, e.g., [6,19,20]) reflects the increasing energy disbalance among helical components and enhancement of one of them. Since this disbalance decreases with growing k according to Eq. (7), generation of helical vortices in a preferential direction occurs in large scales.

Hence, there are three opportunities of helicity manifestation in a turbulent flow:

- $H(k,t)=0$
- $H(k,t)\neq 0$; however, $He_t=\int_{-\infty}^{\infty}H(k,t)dk=0$ is not a helical flow, although it is gyrotropic.
- $H(k,t)\neq 0$ and $He_t\neq 0$—helical turbulence.

Here, in the presence of an external force only, mean helicity appearance and absence is directly connected with the former. Consequently, the spontaneous mirror symmetry violation in a turbulent flow examined in [21], apparently, does not appear. Its appearance is connected with gyrotropic nature of the external impact (including dissipation), because helicity conservation in the Euler flow does not lead to symmetry violation.

By way of example, we examine mean Beltrami flow with $rot\mathbf{u}=\gamma\mathbf{u}$, where γ is a certain pseudo-scalar. This velocity field is a solution of the Euler equation and is often applied for examining flows with the maximal mean helicity. If the velocity field is chosen in the form:

$$\mathbf{u}=U_0(\cos(kz)+\cos(2kz)+\sin(3kz);\ \sin(kz)+\sin(kz)+\cos(3kz);0) \tag{8}$$

its local helicity is

$$\mathbf{u}=kU_0^2(\sin(5kz)+2k\sin(4kz)-3\sin(kz)). \tag{9}$$

On the whole, the total helicity is zero, but the flow comprises regions with different helicities (differing in sign and magnitude).

In the present paper we demonstrate that the external magnetic field and rotation introducing a large-scale mirror symmetry violation into the system lead to the appearance of mean helicity He_t.

1.1. Basic equations

Navier–Stokes equation for a uniform isotropic incompressible flow is:

$$\frac{\partial\mathbf{u}'}{\partial t}+(\mathbf{u}'\nabla)\mathbf{u}'=-\frac{\nabla p'}{\rho}+\nu\Delta\mathbf{u}'+f' \tag{10}$$

$$\nabla\cdot\mathbf{u}'=0 \tag{11}$$

where $\mathbf{u}'$, p', f' are turbulent components of the velocity, pressure and external force, and ρ and ν are the density and kinematic viscosity of the liquid. In a linear approximation, passing into the Fourier space, we obtain:

$$u_i'(k)=\frac{\Pi_{ij}}{(\nu k^2-i\omega)}f_i'(k) \tag{12}$$

where $\Pi_{ij}=\delta_{ij}-k_ik_j/k^2$ is a projection operator. Hence, the velocity field correlator in the Fourier-space is:

$$Q_{ij}(k,\omega)=F\langle u_i'(r)u_i'(0)\rangle=\frac{\Pi_{ij}\Pi_{in}}{(\nu^2k^4+\omega^2)}f_{jn}(k,\omega) \tag{13}$$

where $F\langle\rangle$ is Fourier transformation, and $f_{jn}(k,\omega)$ is a correlator of the field of uniform and isotropic force:

$$f_{ij}(k,\omega)=A(k,\omega)\Pi_{ij}+iC(k,\omega)\varepsilon_{ijk}k_k \tag{14}$$