International Journal of Non-Linear Mechanics 47 (2012) 118–128

Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Coupled thermo-mechanical analysis of shape memory alloy circular bars in pure torsion

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ARTICLE INFO

Article history:
Received 27 April 2011
Received in revised form
12 December 2011
Accepted 26 January 2012
Available online 4 February 2012

Keywords:
Torsion
Shape memory alloy
Thermo-mechanical coupling
Rate-dependent
Ambient conditions

ABSTRACT

Pure torsion of shape memory alloy (SMA) bars with circular cross section is studied by considering the effect of temperature gradient in the cross sections as a result of latent heat generation and absorption during forward and reverse phase transformations. The local form of energy balance for SMAs by taking into account the heat flux effect is coupled to a closed-form solution of SMA bars subjected to pure torsion. The resulting coupled thermo-mechanical equations are solved for SMA bars with circular cross sections. Several numerical case studies are presented and the necessity of considering the coupled thermo-mechanical formulation is demonstrated by comparing the results of the proposed model with those obtained by assuming an isothermal process during loading–unloading. Pure torsion of SMA bars in various ambient conditions (free and forced convection of air, and forced convection of water flow) subjected to different loading–unloading rates are studied and it is shown that the isothermal solution is valid only for specific combinations of ambient conditions and loading rates.

1. Introduction

Shape memory alloys (SMAs) in recent applications are usually subjected to combined loadings in contrast with the early devices that were mostly designed based on using the uniaxial deformation of SMA wires operating as tendons. The recent interest in using sophisticated SMA devices reveals the necessity of analyzing these materials subjected to complicated loadings. There are numerous industrial applications using SMA helical springs as active actuators [17,6]. In addition to actuators, it has been shown recently that due to the hysteretic response of SMAs, helical springs made of these materials can be efficiently used as energy dissipating devices for improving the response of structures subjected to earthquake loads [30]. Speicher et al. [30] studied SMA helical springs subjected to cyclic loads and it was shown that Nitinol helical springs are efficient devices for damping in a vast range of structures besides their ability in minimizing the residual deformations after an earthquake. It is well known that for most practical helical springs (when the ratio of the mean coil radius to the cross section radius is large and the helix angle is small), assuming that each portion of a spring acts as a straight bar under torsion gives accurate results [32]. The application of torsion analysis for studying the behavior of SMA helical springs motivated the authors to seek accurate and efficient analysis techniques for the pure torsion of SMA bars with circular cross sections. There are a few attempts to study the torsion problem for SMA bars in the literature by using numerical methods or considering simplified SMA constitutive relations (see [20,22] for a comprehensive review of the available solutions for torsion of SMA bars). An accurate three-dimensional phenomenological constitutive equation is reduced for studying the one-dimensional pure torsional problem and exact solutions are proposed for analyzing the torsion of straight SMA bars [20] and SMA curved bars and helical springs [22]. In these studies, it is assumed that the material temperature is not affected by the phase transformation and it remains uniformly distributed and equal to the initial temperature during loading–unloading. However, it is well known that the phase transformation in SMAs is accompanied by heat generation during austenite to martensite (forward) and heat absorption during martensite to austenite (reverse) phase transformation [16,2,27]. Assuming a constant temperature is identical to assuming an isothermal process that requires some particular ambient conditions, geometric properties, and loading rates to allow the material to exchange all the phase transformation latent heat with the ambient during loading–unloading. In a recent study, we have shown that in the case of SMA wires and bars subjected to uniaxial loads, response of the material is strongly affected by the thermo-mechanical coupling in SMAs [21]. In Mirzaeifar et al. [21], the heat balance equation that accounts for the phase transformation heat and the heat flux are

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0020-7462/$ - see front matter
doi:10.1016/j.ijnonlinmec.2012.01.007

coupled to the uniaxial constitutive relation of SMAs. Our formulation can be used for SMA wires and bars with circular cross sections subjected to different uniaxial loading–unloading rates in various ambient conditions. The accuracy of the proposed coupled formulation was validated using experimental results. We have shown that the accuracy of assuming adiabatic or isothermal conditions in the tensile response of SMA bars strongly depends on the size and the ambient conditions in addition to the rate-dependency. We concluded that for having an analysis with acceptable accuracy, a coupled thermo-mechanical formulation is inevitable.

To our best knowledge, there is no reported work in the literature on studying the coupled thermo-mechanical response of SMA bars in torsion. In this paper, we present the heat balance equation by considering the phase transformation latent heat and the heat flux effects for bars in pure torsion. This relation is coupled with the exact solution for pure torsion of SMA bars that we presented previously assuming that the SMA bar is under torsion in a constant temperature [20,22]. The generation and absorption of latent heat due to phase transformation and its flux toward the other parts of the cross section in which the material is responding elastically is taken into consideration. Boundary conditions at the outer surface of the bar caused by free or forced convection of air or fluid flow are carefully enforced. For verification purposes the results of the present coupled thermo-mechanical formulation are compared with the experimental data for a thin-walled NiTi tube subjected to pure torsion.

Since the heat conduction inside the SMA bar and the convection with the ambient both are strongly affected by the loading rate, it is shown that the response of SMA bars in torsion is rate dependent. Several case studies are considered and the effect of loading rate and ambient conditions on the torsional response of SMA bars is studied in detail. In each case, the results are compared with the solution obtained by ignoring the thermo-mechanical coupling, and it is shown that the special characteristics of torsion in SMA bars leads to a significant difference between the isothermal and coupled thermo-mechanical results. It is worth noting that although some simplified lumped temperature methods can be used for studying the coupled thermo-mechanical response of SMA bars subjected to uniaxial loading in particular conditions (when there are no propagating transformation fronts along the length, the Biot number is sufficiently small, and thermal boundary conditions at the ends are insulated), due to the non-uniform distribution of shear stress in the cross section of SMA bars subjected to torsion, the effect of temperature non-uniformity and the flux of latent heat is more evident compared to the simple tension without propagating phase fronts.

This paper is organized as follows. In Section 2 a three-dimensional coupled thermo-mechanical formulation for SMAs is briefly explained. The reduction of the model to the one-dimensional pure torsion and the exact solution for SMA bars with circular cross section in torsion is presented in Section 3. Section 4 contains several case studies and some important observations regarding the torsional response of SMA bars for different loading rates and boundary conditions. Section 5 presents a study of the effect of various parameters (loading rate, size, and ambient condition) on the temperature distributions in the cross section. Conclusions are given in Section 6.

2. Coupled thermo-mechanical governing equations for SMAs

Deriving the three-dimensional coupled thermo-mechanical governing equations for SMAs is explained in details in Mirzaeifar et al. [21]. Starting from the first law of thermodynamics in local form and using the second law of thermodynamics, the following coupled energy balance equation is obtained [16]

$$T\boldsymbol{\alpha}:\dot{\boldsymbol{\sigma}}+\rho c\dot{T}+\left[-\pi+T\Delta\boldsymbol{\alpha}:\boldsymbol{\sigma}-\rho\Delta c\ T\ln\left(\frac{T}{T_0}\right)+\rho\Delta s_0 T\right]\dot{\xi}$$
$$=-\mathrm{div}\,\boldsymbol{q}+\rho\hat{g}, \tag{1}$$

where T is the absolute temperature, $\boldsymbol{\alpha},\rho,c$ and s_0 are the effective thermal expansion coefficient tensor, density, effective specific heat, and specific entropy, respectively. The symbols $\boldsymbol{\sigma}$ and T_0 denote the Cauchy stress tensor and reference temperature. It is worth noting that, although the constitutive relations are capable of modeling finite strains [26], we consider the small strains assumption and the formulation is not affected by the stress measure in use. The parameter ξ is the martensitic volume fraction, the terms $\boldsymbol{q}$ and $\hat{g}$ account for the heat flux and any internal heat generation except the phase transformation induced generated heat. Dot on a quantity $(\cdot)$ denotes time derivative and any effective material property $\mathbf{P}$ is assumed to vary with the martensitic volume fraction as $\mathbf{P}=\mathbf{P}^A+\xi\Delta\mathbf{P}$, where the superscript A denotes the austenite phase and the symbol $\Delta(\cdot)$ denotes the difference of a quantity $(\cdot)$ between the martensitic and austenitic phases, i.e. $\Delta(\cdot)=(\cdot)^M-(\cdot)^A$ with M denoting the martensite phase. The term π in (1) is the thermodynamic force conjugate to the martensitic volume fraction and depends on the chosen definition for the free energy. In this paper we use the Gibbs free energy for polycrystalline SMAs [1,26] and this leads to the following thermodynamic force (see [26] for details of deriving this term):

$$\pi=\boldsymbol{\sigma}:\boldsymbol{\Gamma}+\frac{1}{2}\boldsymbol{\sigma}:\Delta\mathbb{S}:\boldsymbol{\sigma}+\Delta\boldsymbol{\alpha}:\boldsymbol{\sigma}(T-T_0)-\rho\Delta c\left[(T-T_0)-T\ln\left(\frac{T}{T_0}\right)\right]$$
$$+\rho\Delta s_0 T-\frac{\partial f}{\partial\xi}-\rho\Delta u_0, \tag{2}$$

where $\boldsymbol{\Gamma}$ is the transformation tensor, $\mathbb{S}$ and u_0 are the compliance tensor and the specific internal energy at the reference state. The function $f(\xi)$ is a hardening function that models the transformation strain hardening in the SMA material. Using this thermodynamic force, the second law of thermodynamics in the form of a dissipation inequality can be written as $\pi\dot{\xi}\geq 0$. This inequality is then used to obtain the conditions that control the onset of forward and reverse phase transformations as

$$\Phi=0,\quad \Phi=\begin{cases}\pi-Y, & \dot{\xi}>0,\\ -\pi-Y, & \dot{\xi}<0,\end{cases} \tag{3}$$

where Y is a threshold value for the thermodynamic force during phase transformation [26]. The consistency during phase transformation guaranteeing the stress and temperature states to remain on the transformation surface is given by $\dot{\Phi}=0$ [28,26]. Substituting (2) and (3) into this consistency condition, the following relation is obtained between the rate of change of martensitic volume fraction, the stress tensor, and temperature

$$\dot{\xi}=-\frac{(\boldsymbol{\Gamma}+\Delta\mathbb{S}:\boldsymbol{\sigma}):\dot{\boldsymbol{\sigma}}+\rho\Delta s_0\dot{T}}{\mathcal{D}^{\pm}}, \tag{4}$$

where $\mathcal{D}^{+}=\rho\Delta s_0(M_s-M_f)$ for the forward phase transformation ($\dot{\xi}>0$) and $\mathcal{D}^{-}=\rho\Delta s_0(A_s-A_f)$ for reverse phase transformation ($\dot{\xi}<0$). The parameters A_s,A_f,M_s,M_f represent the austenite and martensite start and finish temperatures, respectively. Substituting (4) into (1) and assuming $\Delta\alpha=\Delta c=0$ – valid for almost all practical SMA alloys – the following expression is obtained:

$$[T\boldsymbol{\alpha}-\mathcal{F}_1(\boldsymbol{\sigma},T)]:\dot{\boldsymbol{\sigma}}+[\rho c-\mathcal{F}_2(T)]\dot{T}=-\mathrm{div}\,\boldsymbol{q}+\rho\hat{g}, \tag{5}$$

where

$$\mathcal{F}_1(\boldsymbol{\sigma},T)=\frac{1}{\mathcal{D}^{\pm}}(\boldsymbol{\Gamma}+\Delta\mathbb{S}:\boldsymbol{\sigma})(\mp Y+\rho\Delta s_0 T),$$