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Multi-linear stress–strain and closed-form moment curvature response of epoxy resin materials

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A B S T R A C T

A simplified multi-linear stress–strain approach has been used to obtain the closed form nonlinear moment curvature response for epoxy resin materials. The model consists of constant plastic flow in tension and compression. The multi-linear stress–strain model is described by two main parameters in addition to four non-dimensional tensile and six non-dimensional compressive parameters. The main parameters are modulus of elasticity in tension and strain at the proportional elastic limit point in tension. The ten non-dimensional parameters are strain at the ultimate tensile stress, maximum strain, post elastic proportionality stiffness, and post peak strength in the tension model and strain at the proportionality elastic limit, strain at yield strength point, maximum strain, initial elastic stiffness, post elastic proportionality stiffness, and post peak strength in the compression model. Explicit expressions are derived for the stress–strain behavior of the epoxy resins. Closed form equations for moment curvature relationship are presented. The results of tension, compression, and bending tests using digital image correlation technique are presented. Load deflection response of flexural three point bending (3PB) samples could be predicted using the moment curvature equations, crack localization rules, and fundamental static equations. The simulations and experiments reveal that the direct use of uniaxial tensile and compressive stress–strain curves underestimates the flexural response. This model gives an upper bound estimate for flexural over-strength factor.

1. Introduction

Epoxy resins are one of the frequent matrix materials in fiber composites. Mechanical properties (stress–strain relationship) and progressive failure is still a challenge for researchers. Difficulty of a constitutive law in polymer matrix composites is mainly due to the characterization of polymer mechanical behavior. The hydrostatic component of stress has a significant effect on the load deformation response of resins even at low levels of stress. Hydrostatic stresses are known to affect the yield stress of polymers; the absolute value of the yield stress in compression is higher than the ultimate tensile stress. In order to develop a general model for polymer composite materials, the behavior of polymer resins under different types of loading has to be understood. Wineman and Rajagopal [1] used a viscoplasticity model to capture the polymer behavior. Zhang and Moore [2] used the Bodner–Partom internal state variable model originally developed for metals to obtain the nonlinear uniaxial tensile response of polyethylene. By modifying the definitions of the effective stress and effective inelastic strain rate in the Drucker–Prager yield criteria, Li and Pan [3], Chang and Pan [4], and Hsu et al. [5] developed a viscoplasticity approach for the constitutive law of polymer materials. Gilat et al. [6] used an internal state variable model to modify the Bodner model to capture the effects of hydrostatic stresses on the response. In their approach, a single unified strain variable is defined to represent all inelastic strains. Jordan et al. [7] modified the original Mulliken–Boyce model [8] for one dimension to capture the compressive mechanical properties of polymer composites. The original model is a three dimensional strain rate and temperature dependent model for thermoplastic polymers. The majority of the parameters were determined by fitting the model to experimental compressive data. A piecewise-linear tension and compression stress–strain relationship was used to study the mechanical behavior of high performance fiber-reinforced cement composites [9]. Yekani Fard et al. [10,11] studied the nonlinear mechanical behavior of Epon E 863 using the Digital Image Correlation (DIC) system. Hobbiebrunken et al. [12], Bazant and Chen [13], Odom and Adam [14], and Goodier [15] studied the dependency of the failure and strength on the size effect, stress state, and volume of the body subjected to stress in epoxy resin polymers. Giannotti et al. [16] and Vallo [17] used the statistical Weibull analysis approach and estimated the mean flexural strength to be 40% higher than the tensile strength for a Weibull modulus greater than 14.

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Yekani Fard et al. [18,19] used an analytical approach to evaluate the flexural over-strength factor in epoxy resin E 863. They observed that the flexural strength in 3PB beams with groove was at least 14% higher than the tensile peak stress at low strain rates. Flexural over-strength factor is the ratio of the flexural strength (peak stress) to the ultimate tensile strength (UTS).

In this study, the flexural behavior of a beam is investigated in an attempt to establish a relationship between the tensile and compressive stress–strain curves (with constant plastic flow) in one side and moment curvature response of epoxy resin material in the other side. In order to correlate tension, compression stress–strain curves and flexural data, a closed form solution has been developed to obtain moment curvature response. The load deflection response for nonlinear materials under determinate static conditions has been developed. Using inverse analysis, the effect of stress gradient on the multi-linear stress–strain curve obtained from the in-plane uniaxial tests has been studied.

2. Tension and compression multi-linear stress–strain curve

The multi-linear stress–strain curve for tension and compression is bilinear up to the peak stress. Fig. 1 shows the tension and compression stress–strain relationship of epoxy resin materials. The tension and compression curves are defined uniquely by the parameters E, ε_{PEL}, μ_{t1}, μ_{Ut},α, ω, γ, β, ψ, μ_{co}, μ_{c1}, and μ_{Uc}. The tensile stress at the proportionality elastic point (PEL) is related empirically to the stress at the ultimate tensile strength (UTS) point. The ascending part of the tension and compression stress–strain diagrams consist of two linear parts: 0 to PEL, and PEL to UTS in tension or PEL to compressive yield stress (CYS) in compression. The curve after peak strength is idealized as horizontal with σ_{ft} and σ_{fc} as the post peak sustained stress in tension and compression respectively. The constant post peak stress levels (ω and ψ) shows the ability of the model to represent a continuous ($\omega=\psi=1$) or discontinuous stress response. The post peak response in tension terminates at the ultimate tension strain level ($\varepsilon_{Ut}=\mu_{Ut}\,\varepsilon_{PEL}$), and for compression it ends at ultimate compression strain level ($\varepsilon_{Uc}=\mu_{Uc}\,\varepsilon_{PEL}$). In the elastic range, the resin beam in bending could be treated as a bi-modulus material with different moduli in tension and compression. The tension and compression stress–strain relationship are defined as shown in Table 1.

σ_c, σ_t, ε_c, and ε_t are compression and tension stresses and strains, respectively. The ten normalized parameters used in the definition of the constitutive law are defined by

$$\mu_{c0}=\frac{\varepsilon_{PEL,c}}{\varepsilon_{PEL}},\ \mu_{c1}=\frac{\varepsilon_{CYS}}{\varepsilon_{PEL}},\ \mu_{Uc}=\frac{\varepsilon_{Uc}}{\varepsilon_{PEL}},\ \mu_{t1}=\frac{\varepsilon_{Uts}}{\varepsilon_{PEL}},\ \mu_{Ut}=\frac{\varepsilon_{Ut}}{\varepsilon_{PEL}} \tag{1}$$

$$\gamma=\frac{E_c}{E},\ \beta=\frac{E_{PEL,c}}{E},\ \alpha=\frac{E_{PEL,t}}{E} \tag{2}$$

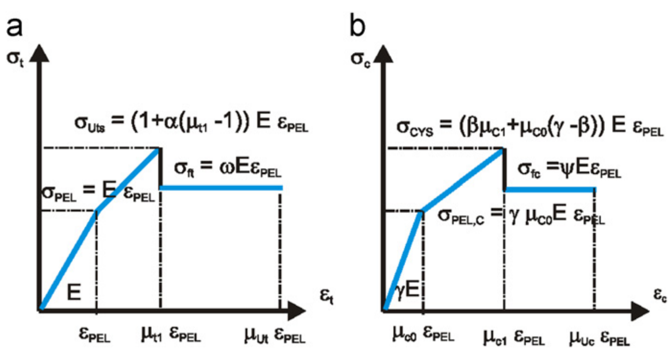

Fig. 1. (a) Constant flow in tension; (b) constant flow in compression.

Table 1
Multi-linear stress–strain curve.

Stress	Definition	Domain of strain
$\sigma_t(\varepsilon_t)$	$E\varepsilon_t$	$0\le\varepsilon_t\le\varepsilon_{PEL}$
	$E(\varepsilon_{PEL}+\alpha\,(\varepsilon_t-\varepsilon_{PEL}))$	$\varepsilon_{PEL}<\varepsilon_t\le\mu_{t1}\,\varepsilon_{PEL}$
	$\omega\,E\,\varepsilon_{PEL}$	$\mu_{t1}\,\varepsilon_{PEL}<\varepsilon_t\le\mu_{Ut}\,\varepsilon_{PEL}$
	0	$\mu_{Ut}\,\varepsilon_{PEL}<\varepsilon_t$
$\sigma_c(\varepsilon_c)$	$\gamma\,E\,\varepsilon_c$	$0\le\varepsilon_c\le\mu_{c0}\,\varepsilon_{PEL}$
	$E\,(\gamma\,\mu_{c0}\,\varepsilon_{PEL}+\beta\,(\varepsilon_c-\mu_{c0}\,\varepsilon_{PEL}))$	$\mu_{c0}\,\varepsilon_{PEL}<\varepsilon_c\le\mu_{c1}\,\varepsilon_{PEL}$
	$\psi\,E\,\varepsilon_{PEL}$	$\mu_{c1}\,\varepsilon_{PEL}<\varepsilon_c\le\mu_{Uc}\,\varepsilon_{PEL}$
	0	$\mu_{Uc}\,\varepsilon_{PEL}<\varepsilon_c$

$$\omega=\frac{\sigma_{ft}}{\sigma_{PEL}},\ \psi=\frac{\sigma_{fc}}{\sigma_{PEL}} \tag{3}$$

Using classical beam theory, linear distribution of strain across the depth is assumed. The stress and strain distribution across a section of a beam with depth h and width b by imposing normalized top compressive strain in different cases are shown in Fig. 2. Normalized heights of compression and tension sub-zones with respect to beam depth h are shown in Table 2. Tables 3 and 4 present the normalized stress at the vertices of the tension and compression sub-zones with respect to tensile stress at the proportionality limit point. The internal force in each compression and tension sub-zone of nine stress distribution cases could be calculated from the stress diagram. The centroid of the stress in each sub-zone represents the line of action and moment arm respect to the neutral axis.

3. Closed-form moment curvature response

The development of stress–strain across the section by increasing the normalized compressive strain is presented in Fig. 3. Stress–strain develops at least to stage 4 where compressive and tensile failure is possible if $\lambda_{max}=\mu_{Uc}$ in case 6, or $\lambda_{max}=F$ in case 4. Moving through different stages in Fig. 3 depends on the controlling value for λ_{max}. Using the auxiliary points defined in Table 5, the transition points defined as tp_{ij} between different stages in Fig. 3 could be presented by the following equations.

$$\begin{aligned}
&tp_{12}=Min(\mu_{c0},A)\\
&tp_{23}=Min(\mu_{c0},C)\ or\ Min(\mu_{c1},B)\\
&tp_{34}=Min(\mu_{Uc},D)\ or\ Min(\mu_{c1},E)\ or\ Min(\mu_{c0},F)\\
&tp_{45}=Min(\mu_{Uc},G)\ or\ Min(\mu_{c1},H)\\
&tp_{56}=Min(\mu_{Uc},I)
\end{aligned} \tag{4}$$

where indexes i and j refer to origin and destination stages, respectively. The net force is obtained as the difference between the tension and compression forces, equated to zero for internal equilibrium, and solved for the neutral axis depth ratio defined as κ. The expressions of net force in some stages result in more than one solution for κ. Using a large scale of numerical tests covering possible ranges of material parameters, the solution of κ which yields the valid value $0<\kappa<1$ was determined and presented in Table 6. Moment expressions are obtained by taking the first moment of the compression and tension forces about the neutral axis. Curvature is calculated by dividing the top compressive strain by the depth of the neutral axis κh. The closed form solutions for normalized moment M_i and curvature φ_i with respect to the values at the tensile PEL points are presented in Eqs. (5)–(7) and Table 6.

$$M=M_{PEL}M'(\lambda,\gamma,\beta,\alpha,\mu_{c0},\mu_{c1},\mu_{t1},\mu_{Ut},\mu_{Uc},\omega,\psi) \tag{5}$$

$$\varphi=\varphi_{PEL}\varphi'(\lambda,\gamma,\beta,\alpha,\mu_{c0},\mu_{c1},\mu_{t1},\mu_{Ut},\mu_{Uc},\omega,\psi) \tag{6}$$