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ScienceDirect
International Journal of Impact Engineering 34 (2007) 377–393
INTERNATIONAL JOURNAL OF IMPACT ENGINEERING
www.elsevier.com/locate/ijimpeng

Elastic wave propagation in cylindrical bars after brittle failures: Application to spalling tests

Teresa Gómez del Río, Jesús Rodríguez Pérez*

Departamento de Ciencia e Ingeniería de Materiales, Universidad Rey Juan Carlos, Tulipán s/n 28933 Móstoles (Madrid), Spain

Received 2 January 2004; received in revised form 24 January 2004; accepted 10 July 2005
Available online 27 September 2005

Abstract

The effect of brittle failures on the elastic wave propagation along cylindrical bars is analysed. From experimental observations provided by spalling tests of ceramic materials, a theoretical analysis is carried out based on a finite elements simulation of the experiments and a mathematical analysis of the pulses by means of Fourier Transform techniques. Differences between the propagating waves before and after the material failure are revealed. After failure, the pulse is influenced by dispersion effects and its shape changes during propagation. To correct this effect, a procedure based on Bancroft's curves is suggested. Finally, some clues about the way to get consistent results from spalling tests are given.

Keywords: Hopkinson bar; Spalling; Brittle material; Dynamic tensile strength; Wave dispersion

1. Introduction

Wave propagation in elastic rods is encountered in many applications in which impacts and fractures under high strain rates take place. The characterization of the dynamic behaviour of materials with the Split Hopkinson Pressure Bar (SHPB) is also based on elastic wave propagation. Apart from material dispersion due to viscoelastic or viscoplastic behaviour,

*Corresponding author. Tel.: +34 91 488 71 59; fax: +34 91 488 81 50.
E-mail address: jesus.rodriguez.perez@urjc.es (J.R. Pérez).

0734-743X/$ - see front matter
doi:10.1016/j.ijimpeng.2005.07.007

Nomenclature

c_0	one-dimensional elastic wave propagation speed
c_n	phase velocity
d	diameter of the bar
DFT	Discrete Fourier Transform
DFT^{-1}	inverse Discrete Fourier Transform
E	Young's modulus
J_0, J_1	Bessel functions of order 0 and 1, respectively
n, k, N, K	integer numbers
R	bar radius
t	time
T	test duration
x	longitudinal coordinate
Δt	interval of time sampling
Δx	distance between two cross-sections
$\Delta\phi_n$	Phase change of a harmonic component of angular frequency ω_n
κ	wavenumber
λ	wavelength
μ, α	Lamé constants
v	Poisson's ratio
ω	angular frequency
ρ	mass density
$\sigma(x,t)$	stress pulse
$\tilde{\sigma}\,(x,\omega)$	Fourier Transform of the stress pulse $\sigma(x,t)$ at cross-section x

geometrical dispersion occurs when the wavelength of the propagating pulses is of the same order as the diameter of the rod (see [1]). Since the analysis of this problem is complex, experimental researchers have taken great care to ensure that one-dimensional wave propagation theory is sufficiently accurate for practical purposes. Two options are available to guarantee the condition that the minimum wavelength is always much greater than the diameter of the rod: (i) the use of small-diameter rods or (ii) to limit the usable frequency range.

The problem of elastic wave dispersion in cylindrical rods is well known. Pochhammer [2] and Chree [3] formulated the equations and solutions for wave propagation in semi-infinite elastic cylindrical rods. They concluded that wave velocity depends on the ratio of the diameter of the bar to wavelength (d/λ) and on the Poisson's ratio. During an impact, non-monocromatic waves are generated. Harmonics with different frequency propagate with different phase velocity altering the wave profile as it travels along the bar. This effect is known as geometrical dispersion. The equation proposed by Pochhammer and Chree is called the frequency equation:

$$\left(2\kappa^2-\frac{\rho\omega^2}{\mu}\right)J_0(\eta R)J_1(\beta R)+4k^2\beta\eta J_1(\eta R)J_0(\beta R)-\frac{2\eta}{R}\frac{\rho\omega^2}{\mu}J_1(\eta R)J_1(\beta R)=0,$$