International Journal of Non-Linear Mechanics 47 (2012) 14–25

Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Instability investigation of creeping viscoelastic flow in a curved duct with rectangular cross-section

M. Norouzi [a], M.R.H. Nobari [b,*], M.H. Kayhani [a], F. Talebi [c]

[a] Mechanical Engineering Department, Shahrood University of Technology, Shahrood, Iran
[b] Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran
[c] Faculty of Engineering, Semnan University, Semnan, Iran

ARTICLE INFO

Article history:
Received 24 August 2010
Received in revised form
16 August 2011
Accepted 17 August 2011
Available online 22 August 2011

Keywords:
Instability
Curved duct
Rectangular cross-section
Viscoelstic
CEF model
Normal stress differences

ABSTRACT

In this paper, instability in the creeping viscoelastic flow inside a curved rectangular duct is investigated numerically for the first time. Using the Criminale–Eriksen–Filbey (CEF) model as the constitutive equation, the governing equations are solved by a second order of finite difference method based on the artificial compressibility algorithm in a staggered mesh. The effects of normal stress differences on the flow stability are investigated. The numerical results obtained indicate that the increase of the negative second normal stress difference of viscoelastic fluid causes stability in the creeping flow in curved ducts, however, the increase of the first normal stress difference intensifies the instability. Furthermore, at the special value of $\Psi_2/\Psi_1 = -0.5$, the interaction of the two normal stress differences results in a stable flow field.

1. Introduction

According to the classical fluid mechanics theories, the fully developed Newtonian flow inside the straight ducts is always rectilinear. In 1956, Ericksen [1] studied the possibility of rectilinear flow for non-Newtonian fluids. He used the Rivlin constitutive equation and found that the viscoelastic flow in straight ducts with non-circular shape of cross section is not rectilinear. For instance, in fully developed viscoelastic flows inside straight ducts with the square cross section, four pairs of counter rotating secondary flows appear in the flow field. He attributed the existence of these secondary flows to the second normal stress difference of viscoelastic materials, which causes an anisotropic behavior in the flow field. A similar structure for the secondary flows is reported by Green and Rivlin [2] for the viscoelastic flow in straight ducts with elliptic cross section. These vortices are also visualized in the experimental observations [3–6]. Fosdick and Serrin [7] improved the results of Ericksen [1] via deriving a precise theorem about the general condition for non-rectilinear flows in non-circular ducts. Truesdell and Noll [8] have captured the corner vortices using the perturbation method. McLeod [9] studied the over determined systems and presented two theorem about the rectilinear steady flow of simple fluids, which cover the results of Fosdick and Serrin [7]. Oldroyd [10] deduced the general conditions for steady viscoelastic flow in straight ducts and explained that the non-zero second normal stress difference and sharp corner presence in the cross section are necessary but not sufficient conditions for the secondary flows existence. He affirmed that secondary flows are not generated in the viscoelastic flows in which the second normal stress difference is proportional to the viscosity. The necessary conditions for the existence of secondary flows in any straight ducts with arbitrary shape of cross section are also affirmed by Huang and Rajagopal [11] to be the non-zero second normal stress difference and shear stress. The general criteria about the direction of corner vortices rotation are recently presented by Yue et al. [12]. There is an interesting analogy between the non-Newtonian laminar flow and Newtonian turbulent flow in straight ducts. Similar conditions for the generation of secondary flows in turbulent flow have been presented by Huang and Rajagopal [13].

Rajagopal and Huilgol [14] studied the rectilinear shear flow of second order fluid between two parallel plates and obtained the upper and lower error bounds for pressure. Rajagopal and Zhang [15] obtained the pattern of secondary deformation in a cylinder under the axial and cross-sectional deformations. They used the perturbation technique by considering the cross sectional deviation from circularity as the perturbation parameter. Mollica and Rajagopal [16] have found the secondary deformations of viscoelastic materials between eccentric cylinders under the pressure gradient. Baldoni et al. [17] investigated the third order fluid flow

* Corresponding author. Tel.: +98 21 64543412; fax: +98 21 66419736.
E-mail address: mrnobari@cic.aut.ac.ir (M.R.H. Nobari).

0020-7462/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2011.08.006

in an eccentric annulus using a perturbation method, where the eccentricity was used as the perturbation parameter and the secondary flows appeared when considering the fourth order perturbation solution. Mollica and Rajagopal [18] studied the axial shearing flow of a third order fluid in an eccentric annulus using a perturbation method and captured the secondary flows as the two pair of counter rotating vortices. Unlike the work of Baldoni et al. [17], they found the secondary flows in the first order term of perturbation solution.

Flow inside a curved channel is one of the basic and important flows in fluid mechanics and it occurs in many engineering and industrial systems such as heat exchangers, piping systems, and so on. Most of the researches in this regard are related to the Newtonian flows and relatively a few numbers have taken into account non-Newtonian fluid flows, especially for viscoelastic fluid flows in curved channels [19–34]. Flow in curved ducts is highly influenced by the effect of reactive centrifugal force resulted from the path curvature. This reactive centrifugal force causes a considerable increase in the radial pressure gradient in the core region of the flow. However, near the inner and outer walls, the axial velocity and reactive centrifugal force tend to zero. To maintain the force balance against the radial pressure gradient, the secondary flows called Taylor–Görtler vortices are generated [35]. Unlike laminar fully developed Newtonian fluid flows inside the straight ducts, fully developed Newtonian flows inside curved ducts are not rectilinear due to the presence of Taylor–Görtler vortices. The intensity of these secondary flows is strong at large Dean numbers and their velocity magnitude is as large as the thirty percent of mean main flow velocity [35]. These vortices increase the flow resistance and the rate of heat transfer in the curved duct. Furthermore, viscoelastic flows inside closed channels are affected by the normal stress differences. As mentioned before, some weak counter rotating vortices can be generated in viscoelastic flows in the straight non-circular ducts, which are resulted from the second normal stress difference and the shape of cross section. The maximum velocity of these vortices is less than 1–3% of the mean main flow velocity [12]. Fan et al. [30] deduced that the first and second normal stress differences have completely reverse effect on the secondary flows in curved ducts. They showed that increasing the first normal stress difference leads to the increment of Taylor–Görtler vortices intensity, while the increment of the negative second normal stress difference weakens the strength of the secondary flows. They attributed these effects to the considerable variation of stress components resulted from the normal stress differences. Also the experimental observations indicate that unlike the viscoelastic flows in the straight pipes, the effect of normal forces on the flow rate in curved pipes is significant and it is possible to decrease the viscoelastic flow drag by polymeric additives.

At small Dean numbers, viscous forces cause the generation of one pair vortices in the flow inside the curved ducts, but at large Dean numbers, viscous effects cannot preserve the secondary flow structure in the form of one pair vortices, therefore, extra vortices known as the Dean vortices [35,36] appear in the flow field. In general, instability can occur in inertial Newtonian flows inside curved channels due to the reactive centrifugal forces resulted from the duct curvature. A few studies have been published concerning with the investigation of non-Newtonian fluid flow instability in curved channels. Shanthini and Nandakumar [37] have carried out the pioneering research on the Dean instability of fully developed Newtonian fluid flow in a curved channel with square cross-section. They have employed power law model as the constitutive equation and have shown that the power change has no effect on the instability structure of Dean vortices, but its decrement leads only to the reduction of the critical Dean number. The study by Joo and Shaqfeh [38,39] has analyzed the stability of Oldroyd-B fluid flow in curved channels. They have proposed an equation for the stability of Oldroyd-B fluid, similar to Orr–Sommerfeld in Newtonian fluid, and have shown that the increment of the elastic property of fluid, Deborah number, can lead to the flow instability. Moreover, some studies have been carried out by Al-Mubaiyedh et al. [40] and Sureshkumar and Avgousti [41] to investigate the effects of temperature and eccentricity on the stability of viscoelastic fluid flows considering the effect of elastic property of the fluid on the Dean instability. Helin et al. [42] and Boutabaa et al. [43] have investigated the effect of elastic property on the Dean instability in MPTT fluid flow in a curved channel with square cross-section. Similar to the previous studies, they have shown that the increment of the elastic property of MPTT can lead to the instability phenomenon whose occurrence is faster than the Newtonian fluid flows. The effect of elastic property on the instability of the Oldroyd-B fluid flow in annular curved channels has been investigated by Robertson and Muller [26]. Fellouah et al. [44] have studied the Dean instability in power law and Bingham fluids in curved rectangular ducts both experimentally and numerically in order to show the effect of yield stress and viscosity dependency to the shear rate on the critical condition. They indicated that the critical Dean number decreases by increasing the power-law index or by decreasing the Bingham number.

In creeping flow, due to the small velocity values, the effects of inertial and reactive centrifugal forces are negligible in comparison with viscous forces. Therefore, creeping Newtonian flows in curved channels are always stable. However, in viscoelastic fluid flows, the effect of elastic properties is able to generate a secondary flow. Bowen et al. [23] have indicated that at large Weissenberg numbers in the creeping flow of the second order fluid flow and the UCM fluids, the intensity of secondary flows in comparison with the main flow is considerable and can affect the flow rate. Norouzi et al. [45] have investigated these effects on the instability of the second order fluid inertial flow in a curved duct with square cross section. They showed that increasing the negative second normal stress difference stabilizes the inertial flow inside the curved duct while increasing the first normal stress difference indicates completely reverse effect. They have also determined the force balance relation of the second order fluid flow in the core region of the flow inside the curved ducts using an order of magnitude technique suitable for the study of the secondary flow generation resulted from the normal stress differences.

In the present work, the instability of creeping viscoelastic fluid flow in a curved channel with rectangular cross-section is investigated numerically for the first time to the best of our knowledge. Here the reversal effects of the first and second normal stress differences on the instability of flow inside curved channels are discovered. Hence, this study is different from the work of Norouzi et al. [45] where the Dean instability in inertial flows inside curved square ducts have been investigated. According to the results of Shanthini and Nandakumar [37], Helin et al. [42], Boutabaa et al. [43], Fellouah et al. [44], and Norouzi et al. [45], change in the rheological properties of non-Newtonian fluids causes only a delay or expedition on the instability. But instability in the creeping viscoelastic flow is caused only by the effect of elastic properties in the absence of reactive centrifugal force. Therefore, it is expected that the structure and form of vortices in the creeping viscoelastic flow become different from those in the inertial flows.

Here, the flow field is considered fully developed and for convenience the cylindrical coordinate is used to analyze the flow in the geometry shown in Fig. 1. The curvature radius of the channel is referred to as $\tilde{R}$ and the dimensions of the channel in the directions of $\tilde{r}$ and $\tilde{z}$ as $\tilde{a}$ and $\tilde{b}$, respectively. A second order finite difference method based on the artificial compressibility has been employed to discretize the governing equations in a