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Rolling cylinder on an elastic half-plane with harmonic oscillations in normal force and rotational speed. Part I: Solution of the partial slip contact problem

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ABSTRACT

We study the effect of harmonic oscillations during the steady rolling of a cylinder on a plane in partial slip contact conditions in the limit of small oscillations. The solution is an extension of that given in Barber et al. [1] for infinitely large coefficient of friction. Here, the effect of varying normal load and hence contact area is investigated in detail by analyzing the first order variation of the tangential force and the corresponding relative displacements.

In particular, the solution is given in terms of an explicit length scale d in the Flamant solution used as a Green function. Appropriate choice of values of d allows to treat both two-dimensional problems and three-dimensional ones having elliptical contact area sufficiently elongated in the direction of the rotation axis.

Also, this analysis can be used as starting point for corrugation calculations in railway tracks, where oscillations in time of the normal forces can result in non-uniform wear and hence in amplification of the corrugation.

1. Introduction

The rolling of cylinders is a classical problem in mechanics. In practice however the cross-section of the roller is never perfectly circular and equivalently, the countersurface on which the motion occurs is never perfectly flat. Here, we shall look at this problem by perturbing a classical solution due to Carter [2] which deals with the traction (or braking) of a cylindrical wheel. His investigation was originally intended to shed light on rail–wheel wear in locomotives but can be applied in other cases, e.g. rolling at the nano-scale [3], to roll-to-roll printing even in modern nanoimprint lithography [4]. However we shall primarily look at the case of rolling for the classical tractive wheel application.

Recently, an elasticity solution for the two-dimensional problem of a rolling cylinder with applied loads having small sinusoidal oscillations superposed to a mean value has been proposed by Barber and Ciavarella [5], Barber et al. [1] and Afferrante [6]. In the first two papers, the full-stick approximation, which corresponds to assume infinitely large friction coefficient, has been adopted to simplify the problem. In particular, in Barber and Ciavarella [5], the transient effects of rolling has been analytically examined using a perturbation technique on the Winkler model in which the surface displacements of the contacting bodies are assumed simply *proportional to the local tractions*. In Afferrante [6], the Winkler model has been extended to finite values of friction coefficient, implying a finite slip zone in the contact region.

The linear perturbation analysis, as explained in Barber et al. [1], implies that the mean contact area semi-width a_0 in the direction of rolling needs to be sufficiently smaller than the wavelength λ of the initial perturbation. In fact Kalker [7,8] has shown that the time for which the system maintains its 'memory' is equal to the time necessary for a point to move from the leading edge to the trailing edge of the contact area.

One important application of models based on perturbation techniques is in the context of studies of corrugation, where there is a sinusoidal forcing in the form of a corrugated profile over which the rolling takes place, and hence there are oscillations of the displacement and the rolling velocity (creepage). Other authors [9–13] have developed similar approaches to study the phenomenon of the corrugation in railway tracks, which results from an unstable interaction between the dynamics of the vehicle and track and the wear mechanism.

Here, we aim to extend the perturbation analysis to a two-dimensional continuum model of a rolling cylinder on a plane

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with finite friction coefficient and for which Carter [2] gave the celebrated classic solution of the steady-state problem. In particular the contact problem, which presents significant mathematical difficulties, is solved in terms of the frequency-dependent receptances defining the tangential forces and displacements.

2. Contact problem

The model under investigation is that developed by Barber et al. [1] and shown in Fig. 1, in which a finite region of slip is present in the contact area as a result of the finite value of the friction coefficient. The vehicle is moving with speed V so that the rolling cylinder is rotating with angular speed Ω. If we superpose a rigid speed V to the system, the cylinder rests and the rail moves with speed V. The torque M opposes the rotation because we are assuming the vehicle is braking. It can be shown that the case of an accelerating vehicle leads to identical expressions for the receptances.

A procedure similar to the usual solution of Cattaneo and Carter's problems, superposing a corrective term to the full sliding solution, is adopted, differently from Barber et al. [1] where a procedure similar to Mossakowski method of integrating flat punch solutions was used—obtaining an Abel equation instead of a standard Cauchy integral, which we develop here.

We concentrate the elastic deformation in the wheel for which we shall use an equivalent modulus and shall treat the rail as rigid. The velocity at a point on the circumference of the wheel can be written as the sum of three contributions:

$$v_x = \left(1+\frac{\partial u_x}{\partial x}\right)\Omega(t)R + \frac{\partial u_x}{\partial t} = \Omega(t)R + V\frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial t} \tag{1}$$

(i) the 'rigid' velocity term $\Omega(t)R$; (ii) the steady-state tensile strain $\partial u_x/\partial x$ which increases the circumference of the wheel and hence the velocity $\Omega(t)R$; (iii) the 'elastic' contribution term $\partial u_x/\partial t$ due to the variation in time of the elastic displacement. Notice we have replaced $\Omega(t)R$ by V in the second term of Eq. (1) since the difference is second order.

We assume linear perturbation for the rotational speed and tangential displacements (now limited to the stick area):

$$\Omega(t) = \Omega_0 + \Omega_1 \exp(\imath\omega t); \quad u_x(x,t) = u_0(x) + u_1(x)\exp(\imath\omega t) \tag{2}$$

as well as loads

$$P = P_0 + P_1 \exp(\imath\omega t); \quad Q = Q_0 + Q_1 \exp(\imath\omega t) \tag{3}$$

which also implies sinusoidal variation of the contact area:

$$a = a_0 + a_1 \exp(\imath\omega t); \quad a_0 = \sqrt{\frac{4P_0R}{\pi E^*}}; \quad a_1 = \frac{\partial a}{\partial P}P_1 = \frac{a_0P_1}{2P_0} \tag{4}$$

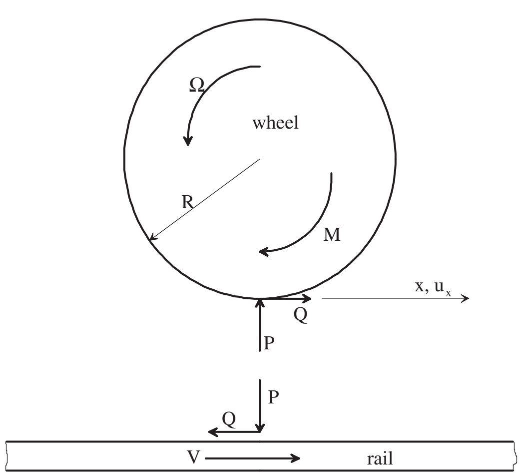

Fig. 1. Model under investigation: rolling cylinder on a rail.

and, in the steady-state, we also have Carter's solution (see [14, Section 12.8]), involving the superposition of two elliptical terms of shear, the full sliding one proportional to the pressure, and a corrective term in the stick region at the leading edge of the contact area:

$$b = b_0 + b_1 \exp(\imath\omega t); \quad b_0 = a_0\sqrt{1-Q_0/fP_0} \tag{5}$$

being b the semi-width of the stick zone. Notice we are assuming uncoupled pressure and shear equations. Coupling is generally neglected since the material in the wheel and rail is elastically nearly identical, and half-plane elasticity is used.

In the stick area the displacements in the tangential direction are as in Section 2 of Barber et al. [1]:

$$u_0(x) = \left(1-\frac{\Omega_0 R}{V}\right)x + C_0 \tag{6}$$

$$u_1(x) = C_1 \exp\left(-\frac{\imath\omega x}{V}\right) + \frac{\imath\Omega_1 R}{\omega} \tag{7}$$

where C_0 and C_1 are constants. We search the solution as a correction q^* in the stick region (c,a) of the full slip solution $q=fp$ everywhere:

$$q(x) = fp(x) - q^*(x) \quad \text{for } x \subset (c,a) \tag{8}$$

where c itself is assumed of the form $c = c_0 + c_1 \exp(\imath\omega t)$.

The correction must be chosen so as to adjust the displacements in the stick region (c,a) to Eqs. (6) and (7) and the corrective shear traction is itself assumed of the form of a perturbation of Carter's solution corrective term $q_0^*(\xi)$:

$$q^*(\xi) = q_0^*(\xi) + q_1^*(\xi)\exp(\imath\omega t) \tag{9}$$

We start from the usual integral [14, Eq. 12.62][1]

$$\frac{du_x}{dx} = -\frac{2}{\pi E^*}\left(\int_{-a}^{a}\frac{fp(\xi)d\xi}{x-\xi} - \int_{c}^{a}\frac{q_0^*(\xi)d\xi}{x-\xi} - \int_{c}^{a}\frac{q_1^*(\xi)\exp(\imath\omega t)d\xi}{x-\xi}\right)$$
$$= \frac{d}{dx}u_0(x) + \frac{d}{dx}u_1(x)\exp(\imath\omega t); \quad c<x<a \tag{10}$$

where the extremes of integration are varying in time, and E^* is the composite modulus:

$$\frac{1}{E^*} = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \tag{11}$$

Of the three integrals, the first two are simple since we know the elliptical form of the integrands [14, par. 12.8.2] which however has p conventionally negative):

$$p(x) = \frac{E^*}{2R}\sqrt{a^2-x^2}; \quad q_0^*(x^*) = \frac{fE^*}{2R}\sqrt{b^2-x^{*2}} \tag{12}$$

where we have changed variables to stay in the centre of stick zone x_c

$$x^* = x - x_c; \quad x_c = \frac{a+c}{2}; \quad b = \frac{a-c}{2} \tag{13}$$

The integrals are

$$\int_{-a}^{a}\frac{fp(\xi)d\xi}{x-\xi} = \frac{fE^*}{2R}\int_{-a}^{a}\frac{\sqrt{a^2-\xi^2}\,d\xi}{x-\xi} = \frac{fE^*}{2R}\pi x; \quad -a<x<a \tag{14}$$

and

$$\int_{c}^{a}\frac{q_0^*(\xi)d\xi}{x-\xi} = \frac{fE^*}{2R}\int_{-b}^{b}\frac{\sqrt{b^2-\xi^2}\,d\xi}{x^*-\xi} = \frac{fE^*}{2R}\pi x^*; \quad -b<x^*<b \tag{15}$$

[1] For the ref system see Barber [14, Fig. 12.3].