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International Journal of Impact Engineering 32 (2006) 1190–1200

INTERNATIONAL JOURNAL OF IMPACT ENGINEERING

www.elsevier.com/locate/ijimpeng

Damage detection of cracked beams based on wavelet transform

Zheng Li*, Shuman Xia, Jun Wang, Xianyue Su

Department of Mechanics and Engineering Science, Peking University, Beijing 100871, People's Republic of China

Received 26 September 2003; accepted 27 September 2004
Available online 16 December 2004

Abstract

In this paper, a damage detection method based on a continuous wavelet transform is proposed and applied to analyze flexural wave in a cracked beam. For flexural waves obtained from FEM or experiments, some useful characters of the incident, reflected and transmitted waves at a certain frequency can be extracted by the Gabor wavelet to exactly identify the damage location and its extent. The orientation of an inclined crack is also discussed in the paper.

Keywords: Continuous wavelet transform; Damage detection; Flexural wave

1. Introduction

Among the existing structural damage detection methods, the stress wave method has been developed into a powerful nondestructive technique, since it is quite sensitive to small defects in structures and easy to perform. Moreover, the wavelet transform (WT) has played an important role in processing wave propagation problems because of its local and self-adaptive time–frequency analytical properties. Nowadays, the nondestructive damage detection methods based on WT mostly focus on analyzing vibration signals in structures and extracting abnormity

*Corresponding author. Fax: +86 10 627 593 78.
E-mail address: lzheng@mech.pku.edu.cn (Z. Li).

0734-743X/$ - see front matter
doi:10.1016/j.ijimpeng.2004.09.012

Nomenclature

a	scaling parameter of wavelet
c_g	group velocity of a flexural wave
$f(t)$	square-integrable function
t	time
τ	translation parameter of wavelet
ω	frequency
$\psi(t)$	mother wavelet function
$\hat{\psi}(\omega)$	Fourier transform of $\psi(t)$
$\overline{\psi(t)}$	complex conjugation of $\psi(t)$
$Wf(a,\tau)$	continuous wavelet transform of $f(t)$
$\psi_G(t)$	Gabor mother wavelet function
$\hat{\psi}_G(\omega)$	Fourier transform of $\psi_G(t)$
$u(x,t)$	displacement function of dispersive flexural wave

characters caused by damage. However, the vibration signals usually have low sensitivity to small damages. Kishimoto et al. [1,2] firstly performed the time–frequency analysis of flexural waves in Euler beam by the Morlet wavelet. Then Tian et al. [3] and Zhang et al. [4] proposed to use this method to identify the damage location in a cracked beam based on numerical and experimental approaches, respectively. Their approaches lead to quite satisfactory results. Using the same approaches, in this paper, the damage extent and crack orientation are studied.

As an example, a cracked cantilever beam under an impact load at its free end is considered. To extract component of a certain frequency from the experimental flexural wave data, the Gabor wavelet, which is better than the Morlet one in the time–frequency analyses, is used. Due to the results of the wavelet transform, the effect of stress wave dispersivity can be overcome and the damage location and its extent can be exactly identified. Furthermore, the effect of the inclined crack can be estimated.

2. Continuous wavelet transform

For any square-integrable function $f(t)$ in the time domain $(-\infty,\ \infty)$, the definition of continuous wavelet transform (CWT) is [5]

$$Wf(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \overline{\psi\left(\frac{t-\tau}{a}\right)} \, \mathrm{d}t, \tag{1}$$

where a and τ denote the parameters of scaling and translation, respectively. In (1), $\psi(t)$ is a mother wavelet and the bar indicates its complex conjugation, and its Fourier transform need to