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Thermally induced buckling of functionally graded hybrid composite plates

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In this paper, thermal buckling analysis is performed on hybrid functionally graded plates (FGPs) with an arbitrary initial stress. The governing equations are derived using the average stress method, including the effect of transverse shear deformation. Then, an eigenvalue problem is formed to evaluate thermal buckling temperatures for simple supported initially stressed ceramic-FGM-metal plates. The effects of functionally graded material (FGM) layer thickness, volume fraction index, layer thickness ratio, thickness ratio, aspect ratio and initial stress on the thermal buckling temperature of hybrid FGPs are investigated. The results reveal that the volume fraction index, initial stresses and FGM layer thickness have significant influence on the thermal buckling of hybrid FGPs.

1. Introduction

In virtue of the high ratio of strength to weight, composite materials have been widely and successfully used in many engineering structures. However, traditional composite materials cannot bear the high temperature, because they have a mismatch of properties across the interface of two bonded layers, which may result in delaminating at high temperature. Thus, a hybrid composite material called FGM which is usually made of the ceramic and metal, has been proposed because it possesses both the high-temperature–resistance property of ceramic and the high toughness of metal. Unlike traditional composite materials, the material properties of an inhomogeneous FGM are smoothly and continuously varied from one constituent to the other, so an FGM is capable of retaining its structural integrity while used at high-temperature environments. Hence, FGMs have been introduced and applied to nuclear reactors, chemical plants and many other industrial fields, where heat-resistant materials are required. When applying FGMs at high-temperature circumstances, the thermal buckling is one of the major concerns to engineers. In this study, the sandwich ceramic-FGM-metal plate is considered instead of all-FGM plate. Since the properties of hybrid FGM can be adjusted by varying not only the compositions of ceramic and metal in FGM layer, but also the thickness of pure-ceramic and pure-metal layer; this make hybrid FGM to have a wider range of material properties and applications than the pure FGM.

A number of investigations dealing with thermal buckling of functionally graded plate had been proposed in the published literature. The nonhomogeneous mechanical properties of the FGP, graded through the thickness, are described by a power function of the thickness variable. Equilibrium equations of a rectangular FGP under thermal loads based on higher order theory were derived by Javaheri [1]. The system of fundamental partial differential equations was established by using the variational method. The derived equilibrium and stability equations for FGPs are identical to the equations for laminated composite plates. A buckling analysis of an FGP under four types of thermal loads was presented. Na [2] investigated the three dimensional thermomechanical buckling of an FGP composed of ceramic, FGM, and metal layers. The thermal buckling behaviors of FGM composite structures due to FGM thickness ratios, volume fraction distributions, and system geometric parameters were analyzed. The thermal buckling of circular functionally graded plate was studied by Najafizadeh [3]. By assuming that the material properties vary as a power form of the thickness coordinate variable z and using the variational method, the buckling of a functionally graded circular plate under various thermal loads was analyzed. Equilibrium equations of a thick rectangular FGP under thermal loads were derived by Lanhe [4]. Thermal loading of uniform temperature rise and gradient through the thickness was considered. The influences of the volume fraction index and the transverse shear on the thermal buckling temperature were discussed.

Shariat [5] presented the thermal buckling analysis of rectangular FGPs with geometrical imperfections based on the classical plate theory. Three types of thermal loading as uniform temperature rise, nonlinear temperature rise through the thickness and axial temperature rise were considered. The thermal buckling of a simply supported skew FGP was investigated by Ganapathi [6]

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based on the first-order theory. Linear and nonlinear temperature rise across the thickness were taken into account. The effects of aspect and thickness ratios, gradient index and skew angle on the critical buckling temperature difference are studied. Morimoto [7] presented the thermal buckling analysis of rectangular FGPs subjected to partial heating in a plane and uniform temperature rise through its thickness. The FGM properties of linear thermal expansion and Young's modulus are changed individually in the thickness direction of the plate with the power law. The effects of material inhomogeneity, aspect ratio, and heated region on the critical buckling temperatures are examined. Thermal buckling of an FGP under the combined effect of elevated temperature and aerodynamic loading was studied by Ibrahim [8]. It is found that the temperature increase has an adverse effect on the FGP flutter characteristics through decreasing the critical dynamic pressure. Decreasing the volume fraction enhances the flutter characteristic. Sohn [9] dealt with the stabilities of FG panels subjected to combined thermal and aerodynamic loads. The first-order theory was used to simulate supersonic aerodynamic loads acting on the panels. The influence of the material constitution of FG panels on thermal buckling and flutter characteristics was examined. The thermal buckling analysis of the square ceramic-metal FGPs with circular holes at the center were presented by Zhao [10]. The effects of the volume fraction index, boundary conditions, hole geometry and hole size on the buckling behavior of FGPs were investigated. Matsunaga [11] presented a higher order deformation theory for thermal buckling of FGPs. By using the method of power series expansion of displacement components, a set of fundamental equations of rectangular FGPs was derived. The critical buckling temperatures of simply supported FGPs were obtained for uniformly and linearly distributed temperatures through the thickness of plates. All previous studies on FGMs were limited to the condition without any initial stress. Meanwhile, the thermal buckling of ceramic-FGM-metal plates has not been seen in the literature.

FGMs are widely applied in engineering structures and unavoidably will encounter problems of structural buckling. The authors' previous studies [12–14] have been focused on the buckling and vibration behaviors of initially stressed FGPs. The results indicate that the existence of an initial stress may significantly affect the behaviors of FGPs. Therefore, to study the thermal buckling of FGPs, the sensitivity of an initial stress is necessary to be considered. In the present paper, the governing equations of hybrid FGPs subjected to non-uniform initial stresses and thermal load are established by using the average stress method, including the transverse shear deformation effect. Then, an eigenvalue problem is formulated for a simply supported initially stressed ceramic-FGM-metal plate to analyze its thermal buckling behaviors. The effects of various variables, such as FGM layer thickness, volume fraction index, layer thickness ratio, thickness ratio, aspect ratio and initial stress, on the thermal buckling temperature of hybrid FGPs are investigated and discussed.

2. Modeling of functionally graded material

A hybrid FGP with uniform thickness h, which is made of ceramics and metals, is considered in this study. Similar to a sandwich laminate plate, the ceramic-FGM-metal plate consists of three layers, a top layer of all-ceramic material ($h/2 \sim z_C$), a core layer of all-FGM material ($z_C \sim -z_M$) and a bottom layer of all-metal material ($-z_M \sim -h/2$). The FGM layer is also made of ceramic and metal, in which the ceramic volume fraction varies from 0% at the interface between the pure-metal layer and FGM to 100% at the interface between the FGM and pure-ceramic layer. Young's modulus E and thermal expansion coefficient α of an FGM layer of the functionally graded plate are assumed as

$$\begin{aligned} E(z) &= E_C V_C + E_M V_M = E_C V_C + E_M(1-V_C) \\ \alpha(z) &= \alpha_C V_C + \alpha_M V_M = \alpha_C V_C + \alpha_M(1-V_C) \end{aligned} \tag{1}$$

here, E_C and E_M are the elastic modulus of the ceramic and metal, respectively. α_C and α_M represent the respective thermal expansion coefficient of the ceramic and metal. V_C is the volume fraction of the ceramic and is expressed by a simple power law as follows:

$$V_C = \left(\frac{2z+h}{2h}\right)^p \tag{2}$$

where the non-negative exponent p is called volume fraction index; z is the thickness coordinate variable, $-z_M \le z \le z_C$. By substituting Eq. (2) into (1), the material properties of an FGM layer can be expressed as

$$\begin{aligned} E(z) &= (E_C - E_M)\left(\frac{2z+h}{2h}\right)^p + E_M \\ \alpha(z) &= (\alpha_C - \alpha_M)\left(\frac{2z+h}{2h}\right)^p + \alpha_M \end{aligned} \tag{3}$$

Eq. (3) can be used to determine Young's modulus E and the thermal expansion coefficient α of a ceramic-FGM-metal plate at any position.

3. Theoretical formulations

An initially stressed hybrid FGP, which is in static equilibrium and subjected to an incremental deformation, is investigated. Following a similar technique described by Brunelle [15] and Chen [12], the governing equations are derived based on a perturbation technique. For a non-uniform initially stressed body, governing equations in static equilibrium can be expressed as

$$(\sigma_{ij}\overline{u}_{s,j})_{,i} + [\overline{\sigma}_{ij}(\delta_{js} + u_{s,j} + \overline{u}_{s,j})]_{,i} + \overline{F}_s + \Delta F_s = 0 \tag{4}$$

where σ_{ij}, $\overline{\sigma}_{ij}$, $\overline{F}_s$ and ΔF_s are the initial stress, perturbing stress, body force and perturbed body force, respectively. u_s and $\overline{u}_s$ represent the initial and incremental displacements.

The incremental displacements are assumed to be of the following forms:

$$\begin{aligned} \overline{u}_x(x,y,z) &= u_x(x,y) + z\varphi_x(x,y) \\ \overline{u}_y(x,y,z) &= u_y(x,y) + z\varphi_y(x,y) \\ \overline{u}_z(x,y,z) &= w(x,y). \end{aligned} \tag{5}$$

The constitutive law including the thermal effect can be written in the following form:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} - \alpha\Delta T \\ \varepsilon_{yy} - \alpha\Delta T \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}. \tag{6}$$

The stress–displacement relations are as follows:

$$\begin{aligned} \sigma_{xx} &= C_{11}(u_{x,x} + z\varphi_{x,x} - \alpha T) + C_{12}(u_{y,y} + z\varphi_{y,y} - \alpha T) \\ \sigma_{yy} &= C_{12}(u_{x,x} + z\varphi_{x,x} - \alpha T) + C_{22}(u_{y,y} + z\varphi_{y,y} - \alpha T) \\ \sigma_{yz} &= C_{44}(\varphi_y + w_{,y}) \\ \sigma_{xz} &= C_{55}(\varphi_x + w_{,x}) \\ \sigma_{xy} &= C_{66}(u_{x,y} + z\varphi_{x,y} + u_{y,x} + z\varphi_{y,x}) \end{aligned} \tag{7}$$

where the stiffness coefficients C_{ij} can be determined.