International Journal of Non-Linear Mechanics 44 (2009) 1048 – 1055

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Steady mixed convection stagnation-point flow of upper convected Maxwell fluids with magnetic field

M. Kumari [a,*], G. Nath [b]

[a] Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India
[b] C/o Dr. S.K. Sinha, KNIT Campus IV/17, KNIT, Sultanpur 228118, India

ARTICLE INFO

Article history:
Received 23 September 2008
Received in revised form 10 August 2009
Accepted 13 August 2009

Keywords:
Mixed convection
Stagnation-point flow
Maxwell fluid
Magnetic field
Boundary layer

ABSTRACT

The steady MHD mixed convection flow of a viscoelastic fluid in the vicinity of two-dimensional stagnation point with magnetic field has been investigated under the assumption that the fluid obeys the upper-convected Maxwell (UCM) model. Boundary layer theory is used to simplify the equations of motion, induced magnetic field and energy which results in three coupled non-linear ordinary differential equations which are well-posed. These equations have been solved by using finite difference method. The results indicate the reduction in the surface velocity gradient, surface heat transfer and displacement thickness with the increase in the elasticity number. These trends are opposite to those reported in the literature for a second-grade fluid. The surface velocity gradient and heat transfer are enhanced by the magnetic and buoyancy parameters. The surface heat transfer increases with the Prandtl number, but the surface velocity gradient decreases.

1. Introduction

During the past century, many engineering problems of fluid mechanics have been solved by using the boundary-layer theory and the results compare well with the experimental observations for Newtonian fluids [1]. An extension of the boundary layer theory to non-Newtonian fluids is found to be rather difficult [2–4]. This difficulty is caused by the diversity of non-Newtonian fluids in their constitutive behaviour and simultaneous viscous and elastic properties. Consequently, most studies on non-Newtonian boundary layers have used simple rheological models such that these two effects can be taken into account separately. In spite of the deficiency of current boundary layer theories for viscoelastic fluids, the studies made using simple rheological models for viscoelastic fluid show some interesting behaviour [2,5,6] which is not observed for Newtonian fluids.

Some non-Newtonian fluids such as nuclear fuel slurries, liquid metals, mercury amalgams, biological fluids, paper coating, plastic extrusions, lubrication oils and greases have applications in many areas in the presence as well as in the absence of the magnetic field.

Beard and Walters [7] used a regular perturbation technique (with elastic number λ^* as a perturbation parameter) to study the boundary layer flow of viscoelastic fluids in the stagnation-point region of a two-dimensional body. They found that the wall shear stress increases with the fluid's elasticity and the velocity inside the boundary layer exceeds that outside the boundary layer. The studies by Teipel [8], Garg and Rajagopal [9], Pakdemirli and Suhubi [10] and Ariel [11] have shown that the regular perturbation technique may not give satisfactory results for viscoelastic fluids. Also Garg and Rajagopal [9] have pointed out that the sign adopted by Beard and Walters [7] (and many others) for the elastic number should be reversed for the second grade model to comply with thermodynamic constraints [12,13]. The above studies indicate that the use of second-grade model is questionable, since it is good only for slow flows with small levels of elasticity. However, in many practical cases the elasticity number is quite large [14]. Therefore, it is better to use more realistic constitutive equations such as Maxwell, Oldroyed-B, Phan-Thien Tanner and Giesukus [15] to study stagnation-point flows of viscoelastic fluids. Bhatnagar et al. [16] employed Oldroyd-B model to study the elastic boundary layer formed above stretching sheets, whereas Sadeghy and Sharifi [17] and Sadeghy et al. [18] studied Blasius and Sakiadis flows of second-grade and upper-convected Maxwell models, respectively, and observed large difference between their predictions of wall shear and boundary layer thickness. Renardy [19] and Hagen and Renardy [20] presented a general formulation for the boundary-layer flows of Maxwell, Phan-Thien Tanner and Giesukus models and showed that the deviation from Newtonian (or inelastic) behaviour would be more significant if the fluid obeyed the upper-convected Maxwell model. Phan-Thien [21] has obtained

* Corresponding author. Tel.: +91 80 2293 3214/2265; fax: +91 80 2360 0146.
E-mail address: mkumari@math.iisc.ernet.in (M. Kumari).

0020-7462/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.ijnonlinmec.2009.08.002

exact solutions to the plane and axi-symmetric stagnation flows of a Maxwellian fluid with inertia without using the boundary layer approximations. Recently, Sadeghy et al. [22] have considered the two-dimensional stagnation-point flow of viscoelastic fluids using upper-convected Maxwell (UCM) model. The equations of motion are simplified using boundary layer theory which yields a single non-linear third-order ordinary differential equation. They solved this equation by spectral method and found that the boundary layer thickness increases and the wall shear stress decreases as the elastic number increases. Hayat and Sajid [23] have extended the analysis of Sadeghy et al. [18] to include the effect of the magnetic field. The dimensionless equation governing the flow problem was analytically solved by homotopy analysis method. Also, Rao and Rajagopal [24] have given a new interpretation of the classical Maxwell model. They have shown that the upper-convected Maxwell (UCM) model can be obtained from the standard KBKZ model. The stored energy that leads to the UCM model is similar to that for a neo-Hookean solid integrated over all past configurations, but weighted by an exponentially decaying function. The UCM model can also be considered as an approximation of a generalized Maxwell model in the limit of small elastic deformations.

The studies reported above deal with flow problem only. The heat transfer problem of a Maxwellian fluid is also important. It is interesting to know the effect of the elasticity of the fluid on the heat transfer rate. It is known that the magnetic field enhances the velocity gradient and heat transfer rate at the surface due to the increase in the Lorentz force. If the temperature difference between the body and the fluid is large, the effect of the buoyancy force is also significant. Hence, the simultaneous effects of elasticity of the fluid, magnetic field and buoyancy force (assisting and opposing flows) on the two-dimensional stagnation flow is an interesting problem, since the heat transfer rate is maximum at the stagnation point.

In this paper, the steady mixed convection flow of viscoelastic fluids which obey the upper-convected Maxwell (UCM) model in the stagnation-point region of a two-dimensional body with applied magnetic field is studied. Both heated and cooled isothermal surfaces have been considered to study the effects of aiding and opposing buoyancy flows. Boundary layer theory is applied to simplify the equations of fluid motion, induced magnetic field and energy. By appropriate transformations, the governing equations are reduced to non-linear coupled ordinary differential equations which are then solved by a finite-difference scheme. The results are compared with those of Beard and Walters [7], Phan-Thien [21] and Sadeghy et al. [22].

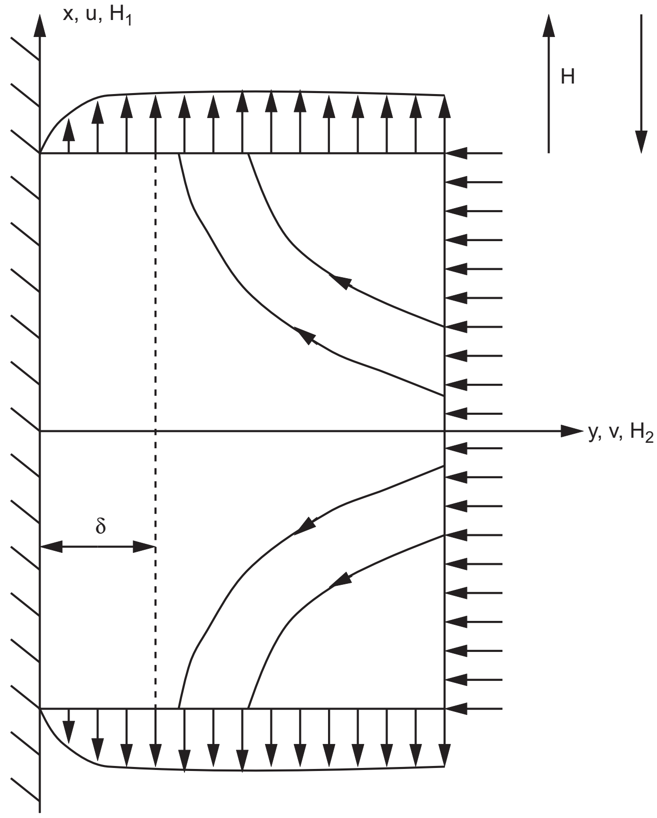

Fig. 1. Physical model and coordinate system.

2. Problem formulation

Let us consider the steady mixed convection flow of an upper-convected Maxwell (UCM) fluid in the stagnation region of a double-infinite vertical surface (see Fig. 1). The magnetic field H is applied in x-direction far away from the surface and it varies with the stream-wise distance x (i.e., $H = H_0(x/L)$, where H_0 is the value of H at $x = 0$ and L is the characteristic length). The fluid Reynolds number Re_x ($= Ux/v$, where U is the velocity at the edge of the boundary layer and v is the kinematic viscosity) and the magnetic Reynolds number Rm_x ($=Ux/\alpha_1^*$, where α_1^* is the magnetic diffusivity) are assumed to be large enough for momentum, thermal and magnetic boundary layers to have developed. The effect of the induced magnetic field is considered here. It is assumed that the normal component (y-component) of the induced magnetic field H_2 vanishes at the wall and the parallel component (x-component) H_1 approaches its given value H at the edge of the boundary layer [25]. The fluid is assumed to be electrically conducting and the surface as non-conducting. Hence, no surface current sheet occurs. In other words, the x-component of the induced magnetic field is continuous across the surface. This condition is expressed by $\partial H_1/\partial y = 0$ when $y = 0$ [25]. The temperature at the wall and in the free stream is assumed to be constant. The effects of viscous dissipation, Ohmic heating and Hall current are not included in the analysis, since they are generally small in the stagnation-point region. All the fluid properties are assumed constant except the density. Under the foregoing assumptions along with Boussinesq appxoximation the boundary layer equations based on the conservation of mass, momentum and energy governing the mixed convection flow can be expressed as [22,25,26]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \tag{2}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= U\frac{dU}{dx} - \frac{\mu_0}{\rho}H\frac{dH}{dx} + v\frac{\partial^2 u}{\partial y^2} \\ &+ \lambda^*\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\,\partial y}\right) \\ &+ \frac{\mu_0}{\rho}\left(H_1\frac{\partial H_1}{\partial x} + H_2\frac{\partial H_1}{\partial y}\right) + g\beta(T - T_\infty), \end{aligned} \tag{3}$$

$$u\frac{\partial H_1}{\partial x} + v\frac{\partial H_1}{\partial y} - H_1\frac{\partial u}{\partial x} - H_2\frac{\partial u}{\partial y} = \alpha_1^*\frac{\partial^2 H_1}{\partial y^2}, \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{5}$$