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On the flow in a uniformly porous pipe

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Abstract

This article considers fully laminar flow of an incompressible viscous fluid in a uniformly porous pipe with suction and injection. An exact solution of the Navier–Stokes equations is given. The velocity filed can be expressed in a series form in terms of the modified Bessel function of the first kind of order n. The volume flux across a plane normal to the flow, the vorticity and the stress on the boundary are presented. The flow properties depend on the cross-Reynolds number, Ua/v, where U is the suction velocity, a is the radius of the pipe and v is the kinematic viscosity of the fluid. It is found that for large values of the cross-Reynolds number, the flow near the region of the suction shows a boundary layer character. In this region the velocity and the vorticity vary sharply. Outside the boundary layer, the velocity and the vorticity do not show an appreciable change.

Keywords: Porous pipe; Suction; Injection; Laminar flow; Exact solution

1. Introduction

An exact solution of the Navier–Stokes equations for flow through a uniformly porous pipe is given. Exact solutions are very important for many reasons. Exact solutions provide a standard for checking the accuracies of many approximate methods such as numerical or empirical. Although computer techniques make the complete integration of the Navier–Stokes equations feasible, the accuracy of the results can be established by a comparison with an exact solution. The exact solution given in this paper is connected with flow of porous boundaries. The flow of fluids over boundaries of porous materials have many applications in practice such as boundary layer control. A simple solution of the Navier–Stokes equations can be obtained for flow over a porous plane boundary at which there is a uniform suction velocity. This solution was found by Griffith and Meredith and was in [1]. There is no solution of the Navier–Stokes equations for flow over a porous plane boundary at which there is a uniform injection velocity. However, if the porous plane is bounded by side walls, a solution of the Navier–Stokes equations can be found for the injection case [2]. The flow between two parallel porous plates with uniform suction at the upper plate and uniform injection at the lower plate has been considered in [3]. For large values of the suction parameter, Ub/v, where U is the suction velocity, b is a characteristic length and v is the kinematic viscosity of the fluid, the flow near the upper plate has a boundary layer character. Velocity varies sharply, the vorticity is concentrated near the suction region and it has a constant value across the channel [3].

The flow of a certain non-Newtonian fluid between two parallel plates with suction and injection has been considered by Jain and Goel [4]. The steady flow of three classes of non-linear fluids of the differential type past a porous plate with uniform suction or injection has been studied in [5]. Steady flow of an Oldroyd-B fluid past a porous plate has been investigated in [6]. Flow of two parallel porous plates rotating with the same angular velocity about an axis was studied by Rajagopal [7]. Flow due to non-coaxial rotation of a porous disc and a fluid at infinity was examined in [8,9]. The flow in a duct of rectangular cross-section with uniform suction and injection has been examined by Mehta and Jain [10], Sai and Rao [11] and Erdoğan [2]. The velocity distribution for the flow in a duct of rectangular cross-section depends on the cross-Reynolds number and

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the aspect ratio. When the suction parameter approaches zero, the velocity reduces to that of the flow in a duct of rectangular cross-section without porous walls. When the aspect ratio approaches zero, the velocity reduces to that for the flow between two parallel porous plates. The vorticity for the flow in a duct of rectangular cross-section with suction and injection has two non-zero components. When the cross-Reynolds number approaches zero, the vorticity reduces to that for the flow in a duct of rectangular cross-section without porous walls. When the aspect ratio approaches zero, the vorticity reduces to that for the flow between two parallel porous plates. For large values of the cross-Reynolds number the variations of the velocity and the vorticity near the region of the suction are sharp and the flow in this region has a boundary layer character. In the other region of the duct, the vorticity remains constant [2].

Fully developed laminar flows through a porous channel and a porous pipe have been investigated by many authors. The first solution for laminar flow in a uniformly porous channel was given by Berman [12]. A simple approximate closed form solution for the flow through a channel and a circular tube with porous walls valid for the entire range of normal injection velocities from zero to indefinitely large was presented in [13]. Laminar flow in a uniformly porous channel was studied in [14]. Hydrodynamic development in a duct with suction has been investigated in [15]. Laminar flow through parallel and uniformly porous walls of different permeability was examined in [16]. The problem of laminar flow through a porous annulus with constant velocity of suction at the walls and with swirl was investigated by Terrril [17]. Laminar flow in a uniformly porous circular pipe with constant suction or injection applied at the wall has been examined and a discussion of previous research has been given in [18]. Fully developed laminar flow with swirl in a porous pipe with injection has been investigated by Terril and Thomas [19]. The incompressible laminar flow along a plane wall with a periodic suction velocity distribution that is transverse to the flow direction has been studied in [20]. A method of velocity turbine blades internally by continuous injection through an interior baffle was analysed in [21]. Flow through a porous channel with fluid injection through one side of the channel was investigated in [22]. The flow connected with porous slider was analysed by Wang [23]. Recently, three-dimensional flow in a porous channel has been studied in [24]. The flows considered in [12–24] are only with injection (or suction) along the channel. Therefore, the results obtained can neither be compared to those of a channel of rectangular cross-section nor to those of a circular pipe with suction and injection.

In this paper, the flow in a porous pipe with injection and suction is considered. An exact solution of the Navier–Stokes equations is given. A complete description of the solution is presented by using the graphs of the velocity, the volume flux across a plane normal to the flow, the vorticity and the stress on the boundary. The velocity, the vorticity and the stress on the boundary depend on only cross-sectional variables; therefore, they do not change along the pipe. The flow considered is affected by a non-dimensional parameter called the cross-Reynolds number (or suction parameter) based on the suction velocity, the radius of the pipe and the kinematic viscosity of the fluid. For large values of this parameter near the suction region, the flow has a boundary layer character. In this region, the velocity varies sharply and the vorticity is concentrated near this region and it does not change appreciably across the pipe.

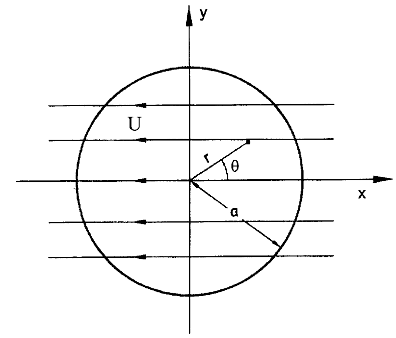

Fig. 1. Flow geometry and coordinate system.

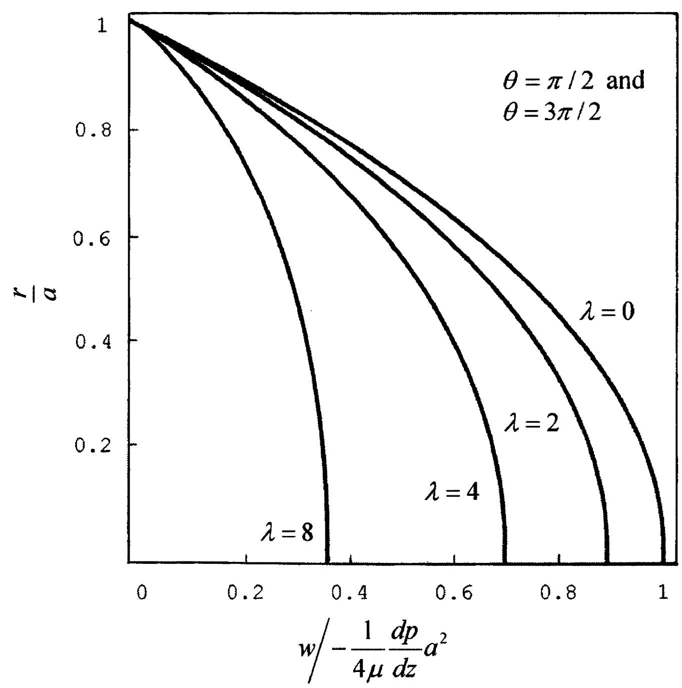

Fig. 2. The variation of $w/-\frac{1}{4\mu}\frac{\mathrm{d}p}{\mathrm{d}z}a^2$ with r/a for various values of λ at $\theta=\pi/2$ and $3\pi/2$.

The velocity distribution for the flow considered is given in a series form in terms of the modified Bessel function of the first kind of order n. When the suction parameter approaches zero, the velocity reduces to that of the flow in a non-porous pipe. For large values of the suction parameter, the series form for the velocity is slowly convergent. But the values of the velocity can be calculated using a computer without difficulty. The variation of the velocity with the non-dimensional distance for various values of the suction parameter is illustrated in Fig. 2. This figure shows that the curves which correspond to $\theta=\pi/2$ and $3\pi/2$ overlap. Fig. 3 denotes the variation of the velocity with the non-dimensional distance for various of λ at $\theta=0$. The curves at $\theta=0$ is differ from those at $\theta=\pi/2$ and $3\pi/2$. The