Journal of Magnetic Resonance 295 (2018) 38–44

Contents lists available at ScienceDirect

Journal of Magnetic Resonance

journal homepage: www.elsevier.com/locate/jmr

Dynamics of magnetization under stimulus-induced rotary saturation sequence

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ARTICLE INFO

Article history:
Received 15 February 2018
Revised 26 June 2018
Accepted 7 July 2018
Available online 20 July 2018

Keywords:
SIRS
Analytical solution
Magnetization dynamics
Bloch equation
MRI

ABSTRACT

We studied stimulus-induced rotary-saturation preparation (which enables measurement of oscillating magnetic fields using MRI) and derived an analytical solution of the Bloch equation to understand magnetization dynamics mathematically and comprehensively and to conduct simulations without sequential-calculation techniques such as the Runge-Kutta method. We formulated the dynamics using the Bloch equation, introducing an additional rotating frame and some approximations to make it into a homogeneous differential equation. Moreover, we found that there are two modes depending on the target oscillating magnetic field. To confirm the validity of the solution, we experimentally investigated its characteristics and performed curve fitting using the analytical model. Considering the constraints on the frame, the analytical solution was found to agree with experimental data. The experimental data indicate that it is necessary to design robust sequences compensating B_0 or $B_{1\mathrm{lock}}$ spatial inhomogeneity to improve measurements. Therefore, experimenters should consider the dynamics of magnetization with RF pulses to rewind the spin phase for accurate measurements.

1. Introduction

At present, blood-oxygenation-level-dependent (BOLD) effects are widely used to visualize neural activity through functional MRI (fMRI). It is easy to specify activation sites in the brain using only T_2^*-weighted images in the resting state and task fMRI [1,2]. However, BOLD effects visualizes hemodynamic processes, and therefore offer limited temporal and spatial resolutions, due to the breadth and slowness of such processes [3]. Recently, new methods of direct measurement of neural activities have been studied, and we have focused on stimulus-induced rotary saturation (SIRS) [4], which can measure oscillating magnetic fields such as those occurring in the brain.

SIRS is a spin-lock sequence, often used to obtain $T_{1\rho}$-weighted or $T_{2\rho}$-weighted images [5]. Such sequences can visualize cartilage tissues with clearer contrast than can T_1- or T_2-weighted images [6–9]. Moreover, SIRS enables us to trace *J*-cupping in O^{17} [10–12], estimate pH [13], and perform other applications.

Functional MRI measurements with low-field MRI scanners using permanent magnets for the static field has been desired from several reasons, such as less expensive maintenance cost. However, BOLD effects cannot be observed with the low field MRI. Therefore, realization of fMRI under low static fields is expected and may contribute to study human brain functions as well as diagnoses of neurological disorders. As described above, the change in MR signals based on the BOLD effects depends on static magnetic field, i.e., larger signal change caused by stronger field [14]. Therefore, SIRS seems a promising fMRI sequence for low-field MRI scanners, since MR signal change based on BOLD effects is negligible, while the signal change obtained by SIRS does not depend on the field strength.

SIRS can also be applied to high-field MRI, but the MR signal changes based on BOLD effects as well as SIRS may be measured simultaneously. One of the methods to suppress BOLD effects is to use an acquisition sequence including 180° pulse, such as the spin-echo sequence. BOLD effects influence T_2^* relaxation, therefore using the 180° refocusing pulse suppresses T_2^* relaxation. On the other hand, by using low-field MRI scanners, these constraints do not exist, so we can use gradient-recalled-echo sequences.

Nagahara et al. have already simulated magnetization dynamics by numerical calculation [15], but they only focused on spin-lock time dependence and frequency characteristics and did not pay attention to the analytical solution.

In this study, we describe the magnetization dynamics using the Bloch equation in differential form, and derive an analytical solution as well. The solution helps us to understand the dynamics

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https://doi.org/10.1016/j.jmr.2018.07.004
1090-7807/

mathematically, such that we can investigate its characteristics and simulate the dynamics and characteristics of SIRS without a sequential-calculation technique such as the Runge-Kutta method. For an example of the former, computational simulations tell us the dynamics under our settings, but the analytical solution can cover comprehensive conditions, including the multiple roots and null points of its transfer function. In addition, this mathematical interpretation is useful for clarifying the limitations of SIRS and improving techniques. It should also allow researchers to avoid computational error, such as accumulation of rounding error.

To derive the solution, we expressed the Bloch equation as a homogeneous linear differential equation by adopting an additional frame and then derived the analytical solution using the Laplace transform. Good agreement was found between the analytical solution and the experimental data. Furthermore, we found that the solution has two modes depending on the strength of a target magnetic field.

2. Theory

SIRS is a preparation-pulse sequence, but not an acquisition sequence. Fig. 1 shows the basic SIRS-pulse sequence, which is composed of three RF pulses: a 90° pulse, a spin-lock pulse, and a −90° pulse. Fig. 2 illustrates the magnetization behavior under SIRS preparation. First, magnetization along the static magnetic field B_0 on the z-axis is flipped onto the $x-y$ plane by a 90° pulse. Second, the spin-lock pulse $B_{1\mathrm{lock}}$ is applied at the same direction as the magnetization whose Larmor angular frequency is the same as the angular frequency ω_{os} of the target oscillating-magnetic field,

$$\omega_{os} = \gamma B_{1\mathrm{lock}}. \quad (1)$$

Here, γ is the gyromagnetic ratio. The magnetization, expressed in rotating frame (x', y', z') as $\boldsymbol{M}' = (M'_x, M'_y, M'_z)$, can receive energy from the target oscillating magnetic field and flip to the $-z'$ direction with precession around the y' axis. This causes M'_y to become smaller than it was without the target magnetic field. At the end of SIRS, the −90° pulse is applied to flip back to z'-axis; then, the gradient magnetic fields are applied to spoil $\boldsymbol{M}'_x$ and $\boldsymbol{M}'_y$. However, the magnetization cannot be thoroughly flipped to the z-axis. We can measure magnetization by measuring the decrement of the longitudinal magnetization as an MR signal.

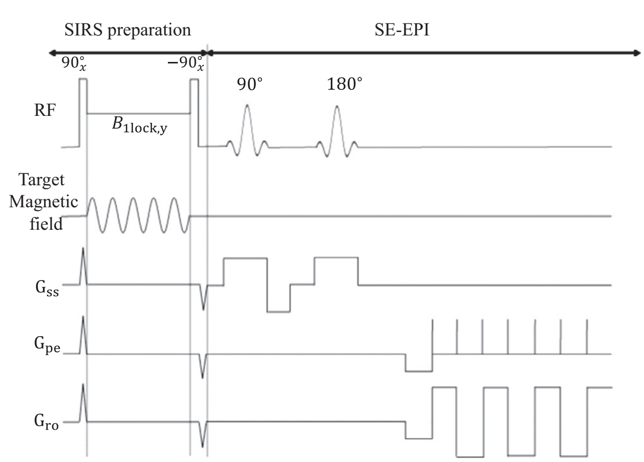

Fig. 1. SIRS preparation and the SE-EPI sequence: this diagram is composed of a pulse sequence and an external target magnetic field. SIRS precedes the SE-EPI acquisition sequence. We applied gradient spoilers before the 90° pulse of SE-EPI to spoil transverse magnetization.

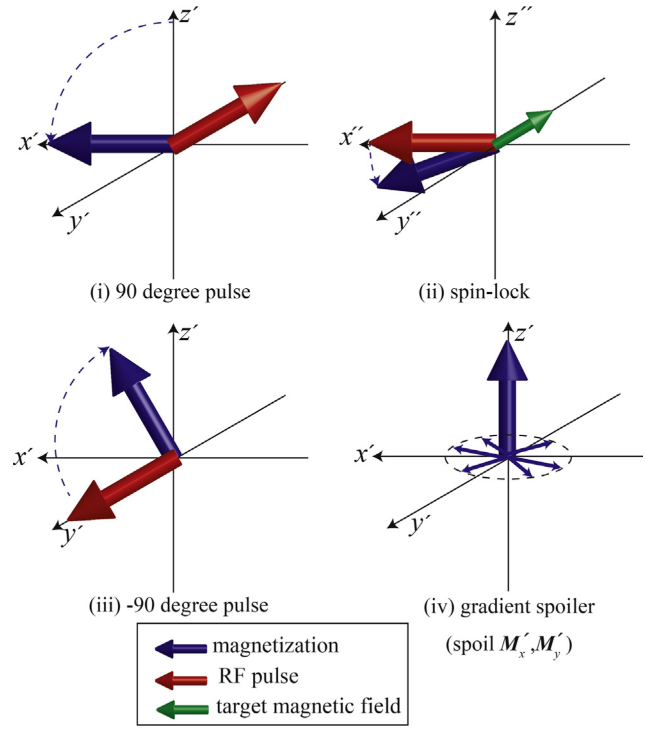

Fig. 2. Overview of the magnetization dynamics under SIRS preparation. The red, green, and blue vectors show the RF pulse, target magnetic field, and magnetization, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3. Material and methods

3.1. Derivation of analytical solutions of the SIRS sequence

First, as Fig. 1 shows, we flip the magnetization to the $x-y$ plane. In the frame of rotation about the z-axis in the laboratory frame, we are able to formulate the magnetization dynamics using the Bloch equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M}' = \gamma \boldsymbol{M}' \times \boldsymbol{B}' - \frac{M'_x \boldsymbol{e}'_x + M'_y \boldsymbol{e}'_y}{T_2} - \frac{(M'_z - M'_\infty)\boldsymbol{e}'_z}{T_1}, \quad (2)$$

where $\boldsymbol{B}', \boldsymbol{e}'_i (i = x, y, z), T_1, T_2$, and M'_∞ are magnetic fields observed from the rotating frame, the basis vector of the rotating frame, the longitudinal relaxation time, the transverse relaxation time, and the magnetization in thermal equilibrium along the z-axis. Below, we call this frame the single rotating frame. Assuming that an RF pulse is applied to the x'-axis and the initial magnetization $\boldsymbol{M}'_0$ is $(M'_{x0}, M'_{y0}, M'_{z0})$, we can obtain

$$\begin{bmatrix} M'_x \\ M'_y \\ M'_z \end{bmatrix} = \boldsymbol{C}' \begin{bmatrix} 1 \\ \frac{1}{\beta'}\exp(-\alpha' t)\sin(\beta' t) \\ \exp(-\alpha' t)\cos(\beta' t) \end{bmatrix}, \quad (3)$$

where

$$\alpha' = \frac{1}{2}(R_1 + R_2), \quad (4)$$

$$\beta' = \sqrt{\omega_x'^2 - \left(\frac{1}{2}(R_1 - R_2)\right)^2}. \quad (5)$$

Here, R_1, R_2, and ω'_x are the longitudinal relaxivity, transverse relaxivity, and $\gamma B'_x$, respectively. $\boldsymbol{C}'$ is the coefficient matrix. We show the elements of $\boldsymbol{C}'$ using the following equations: