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Optimization-based image reconstruction from sparsely sampled data in electron paramagnetic resonance imaging

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ABSTRACT

Electron paramagnetic resonance imaging (EPRI) can yield information about the 3-dimensional (3D) spatial distribution of the unpaired-electron-spin density from which the spatial distribution of oxygen concentration within tumor tissue, referred to as the oxygen image or electron paramagnetic resonance (EPR) image in this work, can be derived. Existing algorithms for reconstruction of EPR images often require data collected at a large number of densely sampled projection views, resulting in a prolonged data-acquisition time and consequently numerous practical challenges especially to *in vivo* animal EPRI. Therefore, a strong interest exists in shortening data-acquisition time through reducing the number of data samples collected in EPRI, and one approach is to acquire data at a reduced number of sparsely distributed projection views from which existing algorithms may reconstruct images with prominent artifacts. In this work, we investigate and develop an optimization-based technique for image reconstruction from data collected at sparsely sampled projection views for reducing scanning time in EPRI. Specifically, we design a convex optimization program in which the EPR image of interest is formulated as a solution and then tailor the Chambolle-Pock (CP) primal-dual algorithm to reconstruct the image by solving the convex optimization program. Using computer-simulated EPRI data from numerical phantoms and real EPRI data collected from physical phantoms, we perform studies on the verification and characterization of the optimization-based technique for EPR image reconstruction. Results of the studies suggest that the technique may yield accurate EPR images from data collected at sparsely distributed projection views, thus potentially enabling fast EPRI with reduced acquisition time.

1. Introduction

Low oxygen concentration, or hypoxia, is a key indicator of tumor resistance to cytotoxic therapies and can be used to guide more effective treatment with radiation and to quantitatively assess or predict treatment response [1]. Electron paramagnetic resonance imaging (EPRI) is considered an effective tool for yielding quantitatively the spatial distribution of tumor-oxygen concentration, referred to as the oxygen image or electron paramagnetic resonance (EPR) image in this work [2]. In pulsed, 3-dimensional (3D) EPRI, the 3D Radon transform can be used to model the imaging process, and existing algorithms such as the standard 3D filtered-backprojection (FBP) algorithm can be used for image reconstruction through inverting the 3D Radon transform [3]. However, existing algorithms often require data collected at a large number of densely sampled projection views, resulting in a prolonged data-acquisition time and consequently practical challenges especially to *in vivo* EPRI [4]. Therefore, there remains a strong interest in shortening data-acquisition time through reducing the number of data samples collected in EPRI. A strategy to shorten the scanning time is to acquire data at a reduced number of sparsely distributed projection views. However, existing algorithms such as the FBP algorithm may yield images with significant artifacts when applied to such sparsely sampled data [5].

Evidence accumulated in recent years reveals that appropriately designed optimization-based reconstruction techniques can yield images with considerably diminished artifacts observed in FBP

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images reconstructed from data collected at sparsely distributed projection views, as demonstrated in computed tomography (CT) [6–12], positron emission tomography (PET) [13], and MRI [14]. Motivated by the successful studies in these imaging modalities, we investigate and develop in this work an optimization-based technique for image reconstruction from data collected at sparse views, thus reducing the scanning time, in EPRI.

In this study, we formulate the EPR image of interest as a solution to a constrained optimization program in which a data divergence in the ℓ_2-norm form is minimized subject to a constraint on the reconstructed image. The specific constraint considered is imposed on the image total-variation (TV), which is the ℓ_1-norm of the gradient magnitude image (GMI) of the image to be reconstructed. We refer to the optimization program as the TV-constrained, data-divergence minimization (TVcDM) program in the work. This type of optimization program has been shown to be effective in yielding images with reduced sampling artifacts that are observed often in FBP reconstruction from data collected at sparsely distributed projection views in CT [7,8,10].

While the TVcDM program is convex, it is non-smooth because the TV constraint considered is non-smooth. Algorithms exist capable of solving convex, non-smooth optimization programs [15,5,16,17], including the Chambolle-Pock (CP) primal-dual algorithm. Previous work suggests that the CP algorithm can be tailored for solving convex optimization programs that arise in various practical imaging problems [17,12]. Therefore, we choose to tailor the CP algorithm to solve the TVcDM program in the context of EPR image reconstruction.

Using computer-simulated EPRI data from numerical phantoms and real EPRI data collected from physical phantoms, we perform studies on the verification and characterization of the optimization-based technique for image reconstruction in EPRI. Results of the studies reveal that the optimization-based technique may yield accurate EPR images from data collected at sparsely sampled projection views and that it can then be exploited for enabling fast EPRI with reduced acquisition time.

The paper starts with an introduction in Section 1, followed by the discussion of the TVcDM program and the tailoring of the generic CP algorithm for solving the program in Section 2. Besides the verification study shown in Section 3, studies with real data collected from physical phantoms are presented in Section 4. Finally, a discussion of the study is given in Section 5.

2. Methods

In this section, we describe the optimization-based technique for image reconstruction in EPRI that consists of a discrete-to-discrete (DD)-data model, TVcDM program, an the CP algorithm tailored to solve the TVcDM program, and the practical convergence conditions on the CP algorithm.

2.1. Continuous-to-continuous (CC)-data model

As mentioned, the data acquisition in EPRI can be modeled by the 3D Radon transform $p(\xi,\theta,\phi)$ of the 3D EPR image $f(x,y,z)$ [18] as

$$p(\xi,\theta,\phi)=\iiint dx\,dy\,dz f(x,y,z)\cdot\delta(x\cos\phi\sin\theta + y\sin\phi\sin\theta+z\cos\theta-\xi), \quad (1)$$

where $\xi\in(-\infty,\infty),\theta\in[0,\pi/2]$, and $\phi\in[0,2\pi]$, and $x\in\mathcal{R},y\in\mathcal{R}$, and $z\in\mathcal{R}$. It can be observed that $p(\xi,\theta,\phi)$ represents the integration of the 3D image function $f(x,y,z)$ over a plane, specified by $\xi=x\cos\phi\sin\theta+y\sin\phi\sin\theta+z\cos\theta$, which is at distance ξ to the origin of the coordinate system, with an orientation specified by unit vector $(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta)^T$ perpendicular to the plane ('T denotes the transpose operation); θ is the angle between the z-axis and the unit vector; ϕ is the angle between the x-axis and the component of the unit vector on the x-y plane; and $\delta(\circ)$ denotes the Dirac function.

Because variables (ξ,θ,ϕ) and (x,y,z) are continuous variables, Eq. (1) is referred to as the continuous-to-continuous (CC)-data model for pulsed EPRI. While the FBP algorithm can be designed for exactly solving $f(x,y,z)$ from full knowledge of $p(\xi,\theta,\phi)$ over angular ranges $\theta\in[0,\pi/2]$ and $\phi\in[0,2\pi]$, it generally requires a large number of measurements of $p(\xi,\theta,\phi)$ densely sampled over the angular ranges to yield numerically accurate reconstruction of $f(x,y,z)$. However, the FBP algorithm yields images with prominent artifacts when it is applied to reconstructing images from data sampled at sparsely distributed projection views.

2.2. Discrete-to-discrete (DD)-data model

Unlike the FBP algorithm that is based upon the CC-data model in Eq. (1), the optimization-based technique considered is based upon a discrete-to-discrete (DD)-data model in which data and image are represented in discrete forms. As discussed below, a DD-data model can be designed based upon the CC-data model.

Assuming that $f(x,y,z)$ has a compact support of size $\xi_{\max}$, we have $\xi\in[0,\xi_{\max}]$. Let $p(\xi_m,\theta_l,\phi_k(\theta_l))$ denote the value of $p(\xi,\theta,\phi)$ sampled at

$$\begin{aligned}\xi_m&=(m-1)\Delta_t\\ \theta_l&=\left(l-\frac{1}{2}\right)\Delta_\theta\\ \phi_k&=\left(k-\frac{1}{2}\right)\Delta_\phi,\end{aligned} \quad (2)$$

where Δ_t,Δ_θ, and Δ_ϕ denote the sampling intervals in $\xi\in[0,\xi_{\max}]$, $\theta\in[0,\pi/2]$, and $\phi\in[0,2\pi]$, respectively; $m\in[1,N_\xi],l\in[1,N_\theta]$, and $k\in[1,N_\phi]$; and $N_\xi=\xi_{\max}/\Delta_\xi,N_\theta=\pi/2\Delta_\theta$, and $N_\phi=2\pi/\Delta_\phi$ the maximum numbers of samples in ξ,θ, and ϕ, respectively.

In this work, we consider a specific sampling scheme characterized by Eq. (2) but with a constraint on the sampling interval Δ_ϕ according to

$$\begin{aligned}\Delta_\phi(\theta_l)&=\frac{2\pi}{N_\phi(\theta_l)}\\ N_\phi(\theta_l)&=\mathrm{round}(2N_\theta\sin\theta_l).\end{aligned} \quad (3)$$

Therefore, the discretization of $\theta\in[0,\pi/2]$ and $\phi\in[0,2\pi]$ based upon Eqs. (2) and (3) yields a set of uniformly distributed sampling points over a solid angular range of 2π, as shown in Fig. 1, with the number of solid angle samples

$$N_a=\sum_{l=1}^{N_\theta}N_\phi(\theta_l). \quad (4)$$

For discussion convenience, we then align the sampled values $p(\xi_m,\theta_l,\phi_k(\theta_l))$ into 1D vector $\mathbf{g}$ of size $J=N_\xi\times N_a$ in a concatenated form in the order of ξ_m,θ_l, and $\phi_k(\theta_l)$ in which entry $j=m+l\times N_\xi+k\times N_\xi\times N_\theta$ is given by the value of $p(\xi_m,\theta_l,\phi_k(\theta_l))$.

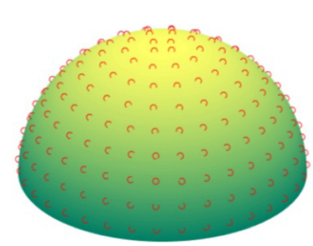

Fig. 1. Uniformly distributed sampling points, indicated by small circles on the hemisphere, over the solid angle range of 2π.