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Non-linear design and control optimization of composite laminated plates with buckling and postbuckling objectives

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Abstract

Multiobjective design and control optimization of composite laminated plates is presented to minimize the postbuckling dynamic response and maximize the buckling load. The control objective aims at dissipating the postbuckling elastic energy of the laminate with the minimum possible expenditure of control energy using a closed-loop distributed force. The layer thicknesses and fiber orientations are taken as design variables. The objectives of the optimization problem are formulated based on a shear deformation theory including the von-Karman non-linear effect for various cases of boundary conditions. The non-linear control problem is solved iteratively until an appropriate convergence criterion is satisfied based on Liapunov–Bellman theory. Liapunov function is taken as a sum of positive definite functions with different degrees. Comparative examples for three-layer symmetric and four-layer antisymmetric laminates are given for various cases of edges conditions. Graphical study is carried out to assess the accuracy of results obtained due to the successive iterations. The influences of the boundary conditions, orthotropy ratio, shear deformation, aspect ratio on the laminate optimal design are elucidated.

Keywords: Structural design and control; Composite laminates; Buckling and postbuckling responses; Non-linear analysis; Shear deformation theory; Various boundary conditions

1. Introduction

Laminated composites are being widely used in many industries, mainly because they exhibit high strength-to-weight and stiffness-to-weight ratios, as well as, the other properties which make them ideally suited for use in weight-sensitive structures. One of the most significant uses of advanced composite materials occurs in the aerospace industry, and particularly, in the construction of large space structures which are built with a high degree of flexibility and mostly with very low natural damping. However, serviceability and safety requirements restrict the allowable limits of the dynamical response to external disturbances to specified values. This problem is commonly known as vibration damping [1].

Design optimization of composite structures is concerned with the best use of the tailoring capabilities of fiber-reinforced laminates to maximize (or minimize) a given design objective. The vibration damping involves the damping out of the excessive vibrations by means of active structural control. In many research studies, the two techniques of the design optimization and active control were treated as separate issues [2–6]. In the last two decades, a great deal of interest for the interaction between these two techniques has been manifested in the literature with a view towards integrating the design optimization and active control in a single formulation [7–12].

In many engineering applications, it is necessary to maximize the buckling load subjected to such design constraints as strength, frequency, displacement, etc. The problem can be formulated as a minimum weight design problem subjected to buckling and other constraints. Alternatively, the laminate may be optimized with respect to several objectives using a multi-criteria design approach. Structures optimized with respect to buckling strength may exhibit low postbuckling resistance [13].

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Thus, for this multiobjective optimization problem, laminates designed with respect to one criterion will perform quite poorly with respect to the other one [14,15]. Therefore, optimization in the postbuckling range becomes an important design consideration for laminates which may be exposed to temperature load or compressive load or combined thermo-mechanical load higher than the buckling load. Many investigators have used integrated approaches to reconciling the conflicting objectives in the design and vibration control of composite structures [16–19]. Further studies on the design and control optimization of composite laminates in the prebuckling and postbuckling ranges can be found in [20–24].

In general, design sensitivity analysis is important to know accurately the effects of the design variable changes on the performance of composite laminates in various cases of boundary conditions. To evaluate these sensitivities efficiently and accurately, it is important to have appropriate techniques associated with good structural models. However, most research studies related to the design and control optimization of composite laminates were carried out based on the classical theories for special cases of boundary conditions. In addition, the optimization problems of composite laminates in the prebuckling range have been extensively studied, but relatively little attention has been directed to such problems in the postbuckling range which need a non-linear (large deflection) analysis.

The current work deals with a non-linear multiobjective optimization problem of composite laminated plates subjected to in-plane compressive forces. The design and control objectives are to maximize the buckling load and to minimize the postbuckling dynamic response with minimum expenditure of control energy. The total elastic energy is taken as a measure of the dynamic response. The fiber orientation angles and layer thicknesses are taken as design variables. The optimization objectives are formulated based on a shear deformation theory including the von-Karmen non-linearity [25]. The optimality condition of Liapunov–Bellman theory [26] is used to obtain the optimal control force and controlled buckled deflection iteratively until an appropriate convergence criterion is satisfied. For this purpose, Liapunov function is taken as a sum of positive definite functions with different degrees. Comparative examples are given for three-layer symmetric and four-layer antisymmetric laminated plates with various cases of boundary condition to show the advantages of the present optimization approach. Also, the influences of boundary conditions, material and geometric parameters on the optimization process are studied.

2. Theoretical formulation and basic equations

Consider a fiber-reinforced composite laminated rectangular plate composed of N anisotropic layers bounded together in an arbitrary lamination scheme such that each layer possesses one plane of elastic symmetry parallel to the mid-plane of the plate. The laminate is of length a, width b, and total constant thickness h; occupying the space $0 \leqslant x \leqslant a$, $0 \leqslant y \leqslant b$ and $-h/2 \leqslant z \leqslant h/2$. The plate is subjected to in-plane compressive forces P_1 and P_2, and the upper surface of the plate ($z = -h/2$) is loaded by a transverse distributed force $q(x, y, t)$ acting as a control force. The present formulation is based on a first-order shear deformation laminate theory accounting for the following Reissner–Mindlin displacements:

$$u_1(x, y, z, t) = u(x, y, t) + z\psi(x, y, t),$$
$$u_2(x, y, z, t) = v(x, y, t) + z\phi(x, y, t),$$
$$u_3(x, y, z, t) = w(x, y, t), \tag{1}$$

where (u_1, u_2, u_3) are the displacements along x, y and z directions, respectively, (u, v, w) are the displacements of a point on the mid-plane, and $\psi$and ϕ are the slope changes in the x and y directions (i.e. rotations about the y- and x-axes), respectively, due to bending. The present study deals with the postbuckling response characterized by finite deformation. Therefore, the strains associated with the displacement (1) must include the geometric non-linear effect. The strains according to the von-Karman theory take the form [25]:

$$\varepsilon_1 = \varepsilon_1^{(0)} + z\psi_{,x}, \quad \varepsilon_2 = \varepsilon_2^{(0)} + z\phi_{,y}, \quad \varepsilon_3 = 0,$$
$$\varepsilon_4 = w_{,y} + \phi, \quad \varepsilon_5 = w_{,x} + \psi, \quad \varepsilon_6 = \varepsilon_6^{(0)} + z\varepsilon_6^{(1)},$$
$$\varepsilon_1^{(0)} = u_{,x} + \tfrac{1}{2}w_{,x}^2, \quad \varepsilon_2^{(0)} = v_{,y} + \tfrac{1}{2}w_{,y}^2,$$
$$\varepsilon_6^{(0)} = v_{,x} + u_{,y} + w_{,x}w_{,y}, \tag{2}$$
$$\varepsilon_1^{(1)} = \psi_{,x}, \quad \varepsilon_2^{(1)} = \phi_{,y}, \quad \varepsilon_6^{(1)} = \phi_{,x} + \psi_{,y},$$

where, a comma denotes partial differentiation with respect to the subscript. On reducing the three-dimensional elasticity problem to a two-dimensional one, the following laminate constitutive equations are obtained:

$$\begin{aligned}&(N_i, M_i, Q_m)\\ &\quad = (A_{ij}\varepsilon_j^{(0)} + B_{ij}\varepsilon_j^{(1)} - P_i, B_{ij}\varepsilon_j^{(0)} + D_{ij}\varepsilon_j^{(1)}, A_{mn}\varepsilon_n),\\ &\quad (i = 1, 2, 6), \quad (m, n = 4, 5).\end{aligned} \tag{3}$$

The quantities N_i, M_i and Q_{mn} are the in-plane force resultants, moments resultants and transverse shear resultants, defined by

$$(N_i, M_i, Q_m) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} (\sigma_i, z\sigma_i, \sigma_m)\,\mathrm{d}z. \tag{4}$$

The laminate stiffnesses A_{ij}, B_{ij} *and* D_{ij} are given by

$$(A_{ij}, B_{ij}, D_{ij}, A_{mn}) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} (\overline{c}_{ij}^{(k)}(1, z, z^2), \overline{c}_{mn}^{(k)}K)\,\mathrm{d}z,$$
$$(i, j = 1, 2, 6), \quad (m, n = 4, 5),$$

where z_k and z_{k-1} are the top and bottom z-coordinates of the kth lamina, σ_i are the stresses, K is a shear correction factor and $\overline{c}_{ij}^{(k)}$ are the stiffnesses of the kth lamina refereed to the problem coordinates. The governing equations of the laminate may be obtained using the dynamic version of the virtual displacement