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Shakedown analysis of beams using nonlinear kinematic hardening materials coupled with continuum damage mechanics

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ABSTRACT

The nonlinear kinematic hardening theory of plasticity based on the Armstrong–Fredrick model and isotropic damage was used to evaluate the cyclic loading behavior of a beam under the axial, bending, and thermal loads. Damage and inelastic deformation were incorporated and they were used for the beam shakedown and ratcheting analysis. The beam material was assumed to follow the nonlinear strain hardening property coupled with isotropic damage. The effect of the damage phenomenon coupled with the elastoplastic nonlinear kinematic hardening was studied for deformation and load control loadings. The Bree's diagram was obtained for two different types of loading, and all numerical results confirmed the reduction of the safe loading domain due to material damage.

1. Introduction

Mechanical elements are often exposed to considerable variations of thermal and mechanical loads, as, for example, variable temperature gradients in the power-generating and processing industry. If the loads history during the lifetime of the considered mechanical elements is not precisely known, the shakedown theory is an appropriate tool for estimating structural safety. In the previous decades, the basic theorems, proposed by Melan [1] and Koiter [2], have been extended to broader classes of problems than initially covered, in order to include changes of temperature, the influence of geometrical changes, and non-linear plastic hardening (see e.g. [3–6]).

Another kind of irreversible dissipative effect preceding eventual structural failure, so-called material damage, has found little attention up to now in the context of shakedown theory. The nonlinear material behavior may be attributed to two distinct material mechanical processes: plasticity and damage mechanics (microcracks, microcavities, nucleation and coalescence, decohesions, grain boundary cracks, and cleavage in regions of high stress concentration). The two degradation phenomena are better described by the theories of plasticity and continuum damage mechanics. Ductile materials usually failed as a result of nucleation, growth, and coalescence of the microdamage. Experimental observations show that the accumulation of the microdamage has a tendency to form macroscopically localized damage, which is a precursor to failure. This progressive physical process of degradation of the material's mechanical properties up to complete failure is commonly referred to as the damage. Variability of damage morphologies has been described in the literature, for example: creep damage, low-cycle fatigue, high-cycle fatigue, and brittle damage [7–10].

With the help of the material softening function [11], Siemaszko [12] presented a step-by-step method of non-shakedown analysis for elastic–plastic discrete structures taking into account nonlinear geometrical effects, nonlinear hardening and ductile damage. Hachemi and Weichert [13,14] extended Melan's theorem to damaging material with linear kinematic hardening in the framework of general continuum mechanics by the concept of effective stresses following the model of Lemaitre and Chaboche [8] and Ju [15].

Two common material models are available to evaluate the cyclic loading behavior of structures, namely isotropic and kinematic hardening theories. The isotropic hardening theory assumes the expansion of the yield surface in stress space and the kinematic hardening theory assumes movement of the yield surface in the stress space along the load direction. Also, different models are available for the kinematic hardening theory, which is responsible for the translation of the yield surface in the stress space. A well-known non-linear kinematic hardening model has been proposed by Armstrong and Frederick [16]. They introduced a kinematic hardening rule containing a recall term which incorporates the fading memory effect of the strain path and essentially makes the rule nonlinear in nature.

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Nomenclature

b	beam width
C	nonlinear kinematic hardening model's constant
D	damage parameter
D_c	interatomic decoherence damage parameter
e	nondimensional total strain
e^p	nondimensional plastic strain
E	elastic modulus
$\tilde{E}$	effective elastic modulus
h	beam height
m	applied moment
M	nondimensional applied moment
$\dot{p}$	incremental accumulated plastic strain
p	applied axial load
P	nondimensional applied axial load
s	nondimensional applied axial stress
S	virgin surface
$\tilde{S}$	resistant effective surface
T	temperature
X	back stress tensor
X'	deviatoric back stress tensor
Y	associate thermodynamics damage variable tensor
α	expansion coefficient
γ	nonlinear kinematic hardening material constant
λ	increment of the plastic multiplier
η	nondimensional depth
ε_p	plastic strain tensor
ε^T	thermal strain tensor
$\varepsilon_p^D, \varepsilon_p^R$	strain damage constants
σ	theoretical axial stress tensor
$\tilde{\sigma}$	effective stress tensor
σ'	deviatoric stress tensor
σ_H	hydrostatic stress
σ_{eq}	equivalent Von–Mises stress
σ_y	yield stress
σ_u	ultimate stress
τ	nondimensional thermal strain tensor
υ	Poisson's coefficient

When continuum damage mechanics are not considered, the nonlinear kinematic hardening of plasticity predicts ratcheting or reverse plasticity for cyclic loading analysis of beams under different types of thermal and mechanical load and combinations of them [17]. Also, the linear kinematic hardening model, including damage, was used to evaluate the cyclic loading analysis of beams [18].

Often, ductile materials undergo a strong plastic deformation, which has a major influence on the damage evolution and vice versa. There are many models with weak coupling between plasticity and damage. The models, which adopt two separate uncoupled damage and plastic loading surfaces with two independent associated flow rules, present a weak coupling between the plasticity and the damage. Those models are used by many authors, extensively [19–21].

Very limited work has been done on damage related to beam bending problems. Krajcinovic [22] did the first study. He applied the damage concept for beam bending problems. Also he defined the isotropic damage variable by means of a parameter called the damage modulus, which is related to the fracture stress. Using this damage model, ultimate moment-carrying capacities are computed for concrete beams, accounting for the shift of the neutral axis. Zhaoxia and Jicheng [23] have developed an isotropic creep damage model applicable for plane and reinforced concrete beams. They defined the damage parameter, which accounts for mortar cracking and interfacial debonding. Combescure and Jiaju [24] used the damage model for analyzing a notched shell in bending. Shi and Voyiadjis [25] have illustrated the use of Lemaitre's damage model for beam and plate bending problems. Chandrakanth and Pandey [26] have developed an exponential damage model for low carbon steel and used it to analyze notched beams to predict their fracture initiation loads. The predicted fracture loads have been compared with experimental results.

The aim of this paper is to investigate the cyclic loading of beams under different types of loading such as thermal, mechanical, and their combinations. The elastic–plastic–damage analysis is performed to study the behavior of the beam structures under load and deformation-controlled conditions. Kinematic hardening theory is used to model the behavior of the beams. During each loading, the damage analysis is performed. An iterative method is proposed to analyze the beam under the cyclic thermal and mechanical loads. Several numerical examples illustrate the influence of material damage in comparison with confirmed results on undamaged structures.

2. Constitutive behavior relations

2.1. Continuum damage mechanics

According to the applied theory of damage mechanics, microscopic change in a material element of surface S causes macroscopic change of the element to its damaged state $\tilde{S}$ due to service condition (Fig. 1), from which the isotropic damage variable D is defined as [8]

$$D = \frac{S - \tilde{S}}{S} \tag{1}$$

D may be considered as an internal state variable characterizing the irreversible deterioration of a material in the thermodynamic sense. Following this theory, the behavior of a damaged material can be represented by the constitutive equations of the virgin material where the usual stress tensor σ is replaced by the effective stress $\tilde{\sigma}$ defined by

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \tag{2}$$

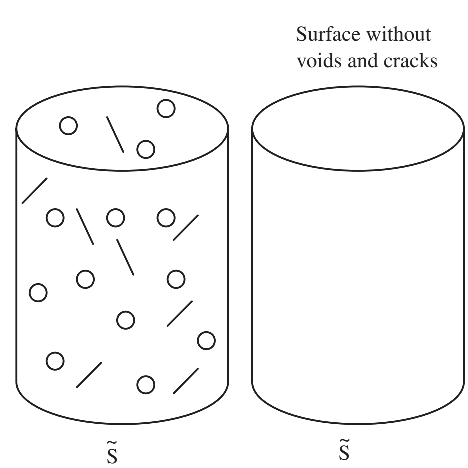

Fig. 1. Definition of the surface (S) and the effective resistant surface ($\tilde{S}$).