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Free vibration and buckling response of hat-stiffened composite panels under general loading

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ABSTRACT

The paper presents finite element free vibration and buckling analysis of laminated hat-stiffened shallow and deep shells using arbitrarily oriented stiffener formulation. Modified approach for modelling the curved stiffener is implemented using necessary transformations. A simplified stiffener formulation is presented to accommodate various shapes of stiffener shapes in developing the rigidity matrix for the finite element formulation. Investigation has been carried out on free vibration and buckling analyses of laminated composite stiffened shell structures with laminated open section (rectangular or 'T' shaped) and closed section ('hat' shaped) stiffeners. Parametric study on the hat-stiffened panels for the free vibration and buckling analyses confirms that the closed section stiffener being torsionally rigid is found to show better performance over open section stiffeners.

1. Introduction

Stiffened plates/shells are widely used as structural components in various fields of engineering. High-performance composite materials are in extensive use due to their inherent high stiffness and strength to weight ratio. Composite stiffened plates and shells are very widely used in aircraft and ship structures to achieve weight saving. Hat shaped stiffeners being torsionally rigid can enhance the strength of a plate/shell structure to a greater extent than open section stiffeners such as rectangular or 'T' section stiffeners.

Very often the structures are subjected to both static and dynamic loads of various magnitudes and complexities. To investigate the actual behaviour of the structures under these loads, rigorous analysis is required to assess the strength and stability under various boundary conditions and loading cases.

The finite element method has emerged as the most powerful numerical tool because of its generality and capability to handle structural and geometrical complexities with ease. In view of this, several commercial software packages have been developed for carrying out structural analyses. However, the commercial software packages suffer from inefficient stiffener modelling which demands that the mesh lines should coincide with the stiffeners. This requirement complicates the analysis of stiffened structures having unequally spaced, arbitrarily oriented stiffeners within the structure. The present investigation is carried out with a generalised finite element formulation of the author [30] on laminated composite stiffened structures which can take care of arbitrarily oriented stiffeners of diverse configurations.

This paper presents an investigation on free vibration and buckling analyses of laminated composite stiffened shell structures with laminated open section (rectangular or 'T' shaped) and closed section ('hat' shaped) stiffeners in particular. Depending upon the behaviour under torsion, the stiffener can be classified into two categories i.e. open and closed sections. The open section such as the 'I', inverted 'T', 'Z', 'J' and rectangular sections are torsionally flexible as against the closed section in the form of hat and box sections. In metallic and FRP construction of aircrafts and ships, the curved section stiffeners have so far been disposed in various ways within the structure. The positioning of the stiffeners with respect to the shell midsurface, i.e. eccentric or concentric is also a matter of concern to the structural analyst. When the stiffener centroid is coincident with the plate/shell midsurface, it is called concentric stiffener, whereas the stiffener centroid and plate/shell midsurface is eccentric, it is called eccentric stiffener. In the present work, emphasis is given in the use of eccentric/concentric, open/closed section laminated stiffeners in laminated shell structures under various loading and boundary conditions.

In the present work, a finite element free vibration and buckling analyses has been carried out of laminated stiffened shells with open and closed section stiffeners following the arbitrary oriented stiffener approach. A shear deformable quadratic eight-noded isoparametric element for the shell in

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Nomenclature

ω	natural frequency
$[K]$	overall elastic stiffness
$[M]$	mass matrix
$[K_G]$	geometric stiffness matrix
$\{\delta\}$	displacement vector
$[L]$	lower triangular matrix
$1/\omega^2$ and $1/\lambda$	eigen values

association with a three-noded curved beam element for the stiffeners has been used in the formulation of laminated stiffened shell analysis. First-order shear deformation theory based on the Reissner–Mindlin theory has been used to take care of the transverse shear deformation. The shell element formulation presented here can be considered as an advance over the degenerated shell concept. The formulation can take care of analysing shallow to deep and thin to thick laminated shells. The stiffener is considered to be a line element whose rigidity has been placed along its centroidal axis. A generalised formulation for the development of the rigidity matrix of open and closed section stiffeners has been presented. Necessary transformations have been incorporated to take care of arbitrarily oriented stiffeners anywhere within the shell element.

2. Literature review

Bhimaraddi et al. [1,2] have presented the finite element analysis of laminated shells of revolution with laminated stiffeners. In their study, they have used 12-noded isoparametric shell element having 64 degrees of freedom in association with four-noded curved beam having 24 degrees of freedom at each node. The analyses suffer from the limitations that the stiffeners have to be placed along the nodal lines only. Mecitoğlu and Dökmeci [3] have analysed the free vibration of a stiffened shallow shell numerically by the collocation method within the framework of the theory of classical thin orthotropic shallow shells. The eccentricity of stiffeners is included in the analysis by providing the moments of inertia of the stiffeners. The vibration characteristics of unidirectional and orthogonally stiffened shallow shells have been studied for various geometrical and material parameters. A parametric study of the vibration characteristics of unidirectional and orthogonally stiffened shallow shells has been carried out for various geometrical and material parameters. Sivadas and Ganeshan [4] have analysed combined shells, cylinder-cone cylinder-plate and stiffened shells for their free vibration characteristics using a higher-order semi-analytical finite element using elements with three nodes and 21 degrees of freedom. Their analysis is based on the shell theory discussed by Kraus [5]. The free vibration characteristics of combined shells and the effect of circumferential stiffeners on frequency has been studied in their analysis. A parametric study on shells with various configurations, material and geometric properties has been carried out. Bert et al. [6] have presented an analytical solution for thin-walled circular cylindrical shells constructed from composite materials and provided with ring or stringer stiffeners. They have made an attempt to evaluate the accuracy of various approximate theories by the use of dimensionless tracer coefficients and assessing the relative improvement in the lowest natural frequency due to the use of ring stiffeners instead of stringer stiffeners. Their study concludes that for a reasonably wide range of geometric parameters, all five theories are in reasonably good agreement. Sinha and Mukhopadhyay [7] have investigated the free vibration of isotropic stiffened shells using a stiffened shallow shell finite element. A high precision curved triangular shallow shell of Cowper et al. [8] has been used to model the shell. Their formulation takes care of the arbitrary orientation of stiffeners, which need not necessarily lie in the direction of the principal curvature of the shell. The element matrices for the stiffened shell element have been generated from the contributions of the shell and the stiffener separately. Examples from the published literature have been solved for the validation of the stiffened shallow shell element. The element suffers from the limitation that it cannot analyse deep shell structures. Liew et al. [9] have presented a survey on the vibration of shell panels which covers a large amount of literature on the subject. Lim and Liew [10] and Liew and Lim [11,12] have presented the p-Ritz method for predicting the vibratory behaviour of shallow cylindrical and doubly curved panels. The method is of an analytic type where the entire domain is represented by admissible functions considering the boundary conditions. Lee and Kim [13] have presented analytical solutions for the free vibrations of the rotating composite cylindrical shells with orthogonal stiffeners using the energy method. Love's shell theory based on the discrete stiffener theory has been used to derive the governing equation of the rotating composite cylindrical shell with orthogonal stiffeners. The stiffeners have been assumed to be an integral part of the shell and have been directly included in their analysis. The effect on the natural frequencies due to the parameters such as the stiffener height-to-width ratio, the shell thickness, and the shell length-to-radius ratio has been studied in their investigation.

Wang and Lin [14] presented a theoretical investigation of stability characteristics of axially compressed simply-supported thin cylindrical shells stiffened by longitudinal stiffeners. Numerical results for cylindrical shells reinforced by only a few longitudinal stiffeners were obtained for the buckling analysis and to describe the buckling modes. Agarwal and Davis [15] have investigated optimum hat-stiffened compression panel designs using structural synthesis technique. The study show a 50% weight savings is possible in using graphite–epoxy panels over optimised aluminium panels apart from the fact that the composite panels are shown to process a variety of proportions at nearly constant weight. William and Steins [16], Funatogawa et al. [17] and Wang and Dawe [18] have made studies on the hat-stiffened plates. Ray [19] has carried out the buckling analysis of laminated hat-stiffened plates using finite element method. Sheinman and Simitses [20] have presented a methodology for the buckling analysis of imperfect, thin, circular cylindrical and isotropic stiffened shells under uniform axial compression. The methodology is based on the smeared technique and the von Karman–Donnell nonlinear kinematic relation in the presence of geometric imperfections. They have demonstrated few examples to draw general conclusions regarding the imperfection sensitivity of stiffened cylinders under axial compression and for various end conditions. Jun and Hong [21] have presented the formulation of the geometrically nonlinear finite element procedure based on an updated Lagrangian description. Cylindrical panels with the general symmetric laminate $[0/\pm\theta/90]_s$ under axial compression are considered and a parametric study is performed in order to investigate the effect of the change of panel width and fibre angles on the buckling stress. The buckling stress was found to vary as