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Effect of axial force on free vibration of Timoshenko multi-span beam carrying multiple spring-mass systems

Yusuf Yesilce [a,*], Oktay Demirdag [b]

[a] Civil Engineering Department, Dokuz Eylul University, Eng. Fac., 35160 Buca, Izmir, Turkey
[b] Civil Engineering Department, Pamukkale University, Eng. Fac., 20070 Kınıklı, Denizli, Turkey

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ABSTRACT

The situation of structural elements supporting motors or engines attached to them is usual in technological applications. The operation of machine may introduce severe dynamic stresses on the beam. It is important, then, to know the natural frequencies of the coupled beam-mass system, in order to obtain a proper design of the structural elements. The literature regarding the free vibration analysis of Bernoulli–Euler single-span beams carrying a number of spring-mass system and Bernoulli–Euler multi-span beams carrying multiple spring-mass systems are plenty, but that of Timoshenko multi-span beams carrying multiple spring-mass systems with axial force effect is fewer. This paper aims at determining the exact solutions for the first five natural frequencies and mode shapes of a Timoshenko multi-span beam subjected to the axial force. The model allows analyzing the influence of the shear and axial force effects and spring-mass systems on the dynamic behavior of the beams by using Timoshenko Beam Theory (TBT). The effects of attached spring-mass systems on the free vibration characteristics of the 1–4 span beams are studied. The calculated natural frequencies of Timoshenko multi-span beam by using secant method for non-trivial solution for the different values of axial force are given in tables. The mode shapes are presented in graphs.

1. Introduction

Extensive research has been carried out with regard to the vibration analysis of beams carrying concentrated masses at arbitrary positions and additional complexities.

Introducing the mass by the Dirac delta function, Chen [1] solved analytically the problem of a simply supported beam carrying a concentrated mass. Chang [2] solved a simply supported Rayleigh beam carrying a rigidly attached centered mass. Dowell [3] studied general properties of beams carrying springs and concentrated masses. Gürgöze [4,5] used the normal mode summation technique to determine the fundamental frequency of the cantilever beams carrying masses and springs. Lin and Tsai [6] determined the natural frequencies and mode shapes of Bernoulli–Euler multi-span beam carrying multiple spring-mass systems. Liu et al. [7] formulated the frequency equation for beams carrying intermediate concentrated masses by using the Laplace Transformation Technique. Wu and Chou [8] obtained the exact solution of the natural frequency values and mode shapes for a beam carrying any number of spring masses. Naguleswaran [9,10] obtained the natural frequency values of the beams on up to five resilient supports including ends and carrying several particles by using Bernoulli–Euler Beam Theory (EBT) and a fourth-order determinant equated to zero. Laura et al. [11] studied on the cantilever beam carrying a lumped mass at the top, obtaining analytical solution and introducing the mass in the boundary conditions. Zhou [12] studied the free vibration of multi-span Timoshenko beams by using Rayleigh-Ritz method. He developed the static Timoshenko beam functions which are composed of a set of transverse deflection functions and a set of rotational angle functions as the trial functions. Wu and Chen [13] performed the free vibration analysis of a uniform Timoshenko beam carrying multiple spring-mass systems by using numerical assembly method. Lin and Chang [14] studied the free vibration analysis of a multi-span Timoshenko beam with an arbitrary number of flexible constraints by considering the compatibility requirements on each constraint point and using a transfer matrix method. Wang et al. [15] studied the natural frequencies and mode shapes of a uniform Timoshenko beam carrying multiple intermediate spring-mass systems with the effects of shear deformation and rotatory inertia. Other studies on the vibration analysis of beams carrying masses are presented in Refs. [16,17]. In structural engineering, this problem is similar to free vibration of an elastic beam on elastic foundation and a pile embedded in soil. Doyle and Pavlovic [18] solved the partial differential equation for free vibration of beams partially attached to elastic foundation using separation of variable method and neglecting shear effect. Catal [19,20] calculated natural frequencies and relative stiffness

* Corresponding author. Tel.: +90 232 4127027; fax: +90 232 4531191.
E-mail address: yusuf.yesilce@deu.edu.tr (Y. Yesilce).

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of the pile due to the values of axial forces acting on the pile and the shape factors. Yesilce and Catal [21] calculated normalized natural frequencies of the pile due to the different values of axial force using carry-over matrix and considering rotatory inertia.

In the presented paper, we describe the determination of the natural frequencies of vibration of the multi-span uniform Timoshenko beam carrying multiple spring-mass systems with/without axial force effect.

2. The mathematical model and formulation

A Timoshenko uniform beam supported by T pins by including those at the two ends of beam and carrying S spring-mass systems is presented in Fig. 1. From Fig. 1, the total number of stations is $M = S+T$. The kinds of coordinates which are used in this study are as: $x_{v'}$ are the position vectors for the stations ($1 \leqslant v' \leqslant M$); x_p^* are the position vectors of the spring-mass systems ($1 \leqslant p \leqslant S$); $\bar{x}_r$ are the position vectors of the pinned supports ($1 \leqslant r \leqslant T$).

From Fig. 1, the symbols of $1', 2', \ldots, v', \ldots, M'-1, M'$ above the x-axis refer to the numbering of stations. The symbols of $1, 2, \ldots, p, \ldots, S$ below the x-axis refer to the numbering of spring-mass systems. The symbols of $(1), (2), \ldots, (r), \ldots, T$ below the x-axis refer to the numbering of pinned supports.

Elastic curve function of Timoshenko beam consists of two components as [19–21]:

$$y(x,t) = y_b(x,t) + y_s(x,t) \tag{1}$$

where $y_b(x,t)$ and $y_s(x,t)$ are elastic curve functions obtained due to bending and shear effects, respectively; x is the beam position; t is time variable.

The total potential energy Π_e of the beam can be stated as:

$$\Pi_e = \frac{1}{2}\int_0^L \left[EI_x \times \left(\frac{\partial^2 y(x,t)}{\partial x^2} - \frac{\partial^2 y_s(x,t)}{\partial x^2} \right)^2 + \frac{AG}{k} \times \left(\frac{\partial y(x,t)}{\partial x} - \frac{\partial y_b(x,t)}{\partial x} \right)^2 - N \times \left(\frac{\partial y(x,t)}{\partial x} \right)^2 \right] \mathrm{d}x \quad (0 \leqslant x \leqslant L) \tag{2}$$

where N is the axial compressive force; L is length of the beam; A is the cross-section area; I_x is moment of inertia; k is the shape factor due to cross-section geometry of the beam; E, G are Young's modulus and shear modulus of the beam, respectively.

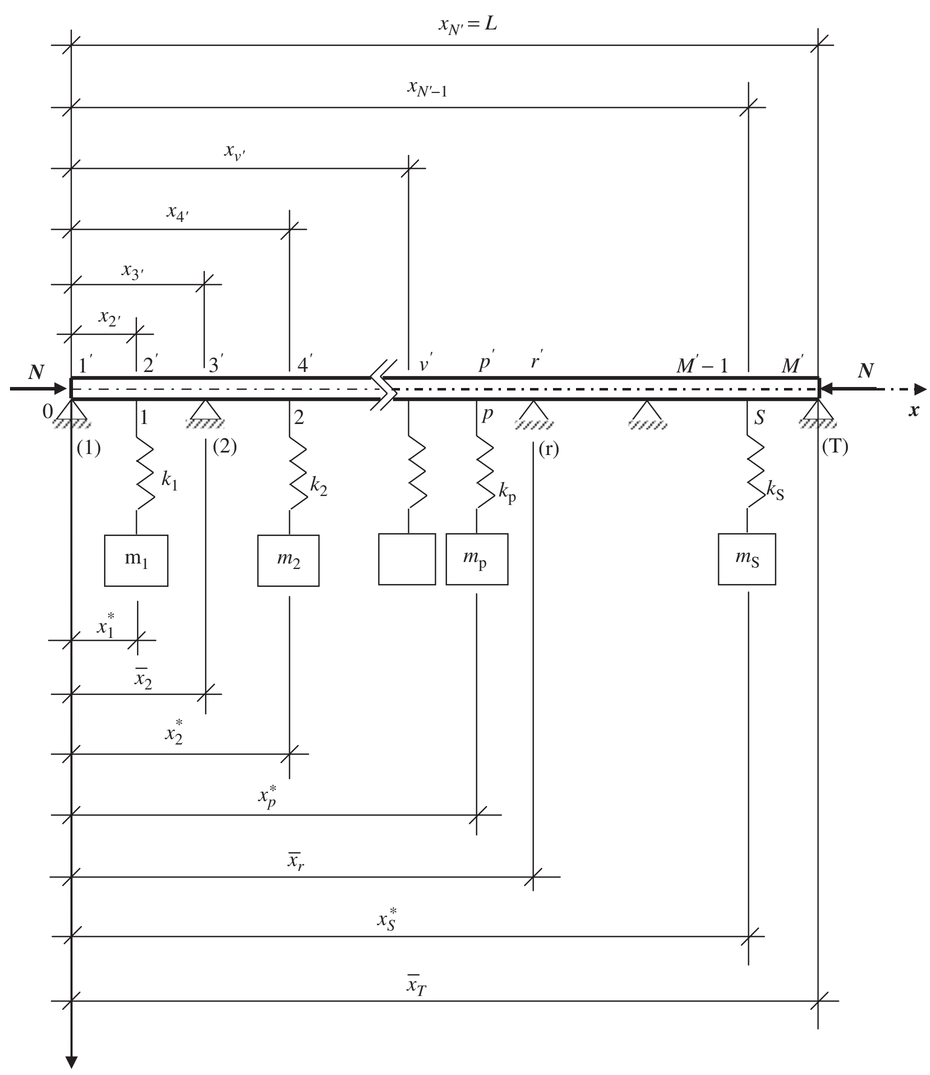

Fig. 1. A Timoshenko uniform beam supported by T pins and carrying S spring-mass systems.