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On thermal instabilities in a viscoelastic fluid

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Abstract

The Bénard–Marangoni thermal instability problem for a viscoelastic Jeffreys' fluid layer bounded above by a realistic free deformable surface and by a plane surface below is investigated using a linear stability analysis. It has been shown that both the relaxation time and surface deflection have a destabilizing effect unlike the retardation time. A point of codimension 2 has been identified which means that instability here takes the form of a competition between stationary and oscillatory convections. When the lower boundary is free but plane, an analytic treatment has identified an oscillatory disturbance with zero critical wavenumber which is not found in the absence of surface deformation.

Keywords: Thermal instability; Jeffreys; Viscoelastic; Bénard–Marangoni; Deformable surface

1. Introduction

Thermal instability of a horizontal layer of fluid heated from below has been extensively investigated in the case of Newtonian fluids [1–11] and to a lesser extent in viscoelastic fluids [12–20]. The recent renewed interest in this area is due to potential applications such as the growth of crystals in a space environment. The two mechanisms responsible for instability are the density variation generated by the thermal expansion of the fluid and the surface tension gradients due to temperature fluctuations at the upper surface of the layer. Investigation of the former mechanism is referred to as the Rayleigh–Bénard problem, while investigation of the latter is called the Marangoni problem.

In this paper we examine the coupled Bénard–Marangoni problem for a Jeffreys viscoelastic fluid layer bounded above by a realistic free deformable surface and by a plane rigid surface below using a linear stability analysis. The approximation that is commonly referred to as the Boussinesq approximation was first developed by Oberbeck [21,22]. It should be noted that it is an approximation not in the classical sense but one which is a combination of terms of different orders. This was discussed in detail by Rajagopal et al. [23]. Interestingly enough the approximation works exceedingly well. Our objective is to investigate in some detail the elastic effects of both relaxation and retardation and in particular surface deflection on stability. Results for the Maxwell fluid are deduced as a special

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case. The corresponding problem for Newtonian fluids was analysed by Benguria and Depassier [24]. Previous works for viscoelastic fluids have considered instability due to buoyancy alone [12–16], surface tension alone [17,18] and combined Bénard– Marangoni convection [19,20], all without surface deflection. Hence, it is this surface deflection that differentiates this work from previous works.

We have utilized a linear stability analysis as a first step towards investigating this problem. The moving upper boundary makes it a challenging problem for non-linear analysis.

It has been shown, inter alia, that both relaxation time and surface deflection have a destabilizing effect unlike the retardation time. A point of codimension 2 has been identified which means that instability here takes the form of a competition between stationary and oscillatory convections.

The paper runs as follows. In Section 2, the problem is clearly stated and the relevant equations of motion given. The linear stability analysis is carried out in Section 3, while Section 4 is devoted to numerical results and their analysis. Section 5 gives a long-wave approximation and the final section summarizes the results.

2. Statement of the problem

A horizontal layer of viscoelastic Jeffreys' fluid is initially at rest between the planes $z = 0$ and $z = d$ and acted upon by gravity. The lower surface is plane rigid and a perfect thermal conductor while the upper

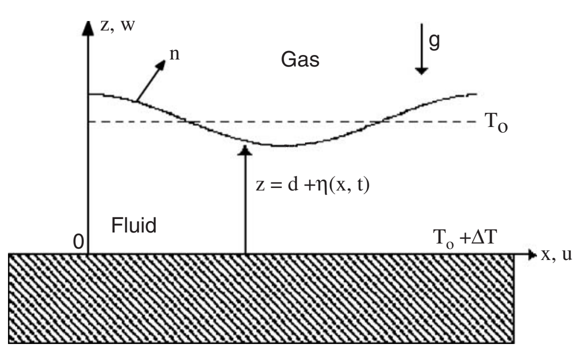

Fig. 1. Jeffreys' fluid with a free deformable surface, heated from below.

surface is free, deformable and in contact with an ambient gas which exerts a constant pressure p_a on it. The latter is a perfect thermal insulator in respect of temperature perturbations. As motion sets in due to the Bénard–Marangoni convection, the free surface is deformed. We work in a Cartesian frame of reference (x, y, z) and assume that the motion is two dimensional (see Fig. 1).

Let the deformable surface be $z = d + \eta(x, t)$. We shall assume the Boussinesq approximation [25] so that the governing equations for the fluid are

$$\nabla \cdot \mathbf{q} = 0, \tag{2.1}$$

$$\rho_0 \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \boldsymbol{\tau}, \tag{2.2}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \kappa \nabla^2 T, \tag{2.3}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{2.4}$$

$$\xi = \xi_0 - \gamma(T - T_0), \tag{2.5}$$

while the constitutive equation for the viscoelastic fluid is [16]

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \boldsymbol{\tau} = \mu \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) [\nabla \mathbf{q} + (\nabla \mathbf{q})^{\mathrm{T}}]. \tag{2.6}$$

Here $\mathbf{q} = (u, 0, w)$ is the velocity, p the pressure, $\mathbf{g}$ the gravitational force, ρ the density, T the temperature, ξ the surface tension, $\boldsymbol{\tau}$ the extra-stress tensor, $\kappa, \alpha, \gamma, \lambda_1, \lambda_2, \mu$ fluid constants representing respectively thermal diffusivity, thermal expansion coefficient, rate of change of surface tension with temperature, the relaxation time, the so-called retardation time, and viscosity, and ρ_0, T_0, ξ_0 are reference values.

3. Linear stability analysis

We first obtain the solution of the above system (2.1)–(2.6) in the static case. Here $\mathbf{q} = 0$ and one can