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Non-linear dynamic response of a thin laminate subject to non-uniform thermal field

Xiaoling He

Department of Mechanical Engineering, University of Wisconsin-Milwaukee, 3200 N Cramer Street, P.O. Box 784, Milwaukee 53201, USA

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Abstract

Non-linear dynamics behavior of a thin isotropic laminate in a simply supported boundary condition is studied for its response with both mechanical and thermal loads in effect. The thermal effects of both the in-plane and transverse non-uniform temperature variations in steady-state are considered. The equation of motion for the laminate deflection is reduced to the Duffing equation in a decoupled modal form by means of a generalized Galerkin's method. The stress field as a function of deflection and temperature variation is also obtained in a plane stress condition for its non-linear elastic behavior with von Karman strain field.

For an exemplary laminated microstructure used as a printed wiring board, it is found that a high rise of the in-plane temperature increases the resonance frequency and could significantly increase the stresses of the lamina. The through thickness temperature variation has no significant effect on the deflection. Failure analysis is also made based on the composite failure criteria for a laminate to identify the critical mechanical and thermal loads.
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Keywords: Non-linear dynamics; Laminate; Duffing equation; Thermal vibration

1. Introduction

The non-linear deformation has to be taken into account when the deflection is over half of the thickness for a thin laminated structure. Non-linear dynamic response of a thin laminated microstructure can be induced by perturbations of initial deflection and velocity, or by the imposed load. In certain applications, both thermal and mechanical load are present. For example, a thin laminated board used in micro-electronics packaging contains both conduction lamina and insulation lamina. The embedded electrical circuits make the in-plane temperature variation non-uniform. In addition, the differences in the interfacial structures and materials cause temperature variation through its thickness. This thermal electrical coupling effect imposes the thermal field, and induces thermal stresses that affect the laminate deformation behavior.

E-mail address: xiaoling@uwm.edu.

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Nomenclature

A	stiffness matrix of the laminate
A^*	inverse matrix of A
A^*_{ij}	elements of A^*
B	coupling rigidity matrix of the laminate
C_{ij}	coefficient of the stiffness matrix C
$(E)^{(k)}$	kth lamina Young's modulus
$F(x, y, t)$	Airy's stress function
$\tilde{F}(x, y, t)$	Airy's stress function with complimentary function
$F_1, F_2, F_{11}, F_{22}, F_{66}$	coefficients of failure function
N	in-plane force vector
N^{T}	thermal force vector
N^{F}	Airy's stress function vector
M	applied moment vector
M^{T}	thermal moment vector
$\mathrm{N}(w)$	function of in-plane force and deflection
$Q^{(k)}_{ij}$	stiffness matrix elements of a laminate in plane stress condition
$R(x, y)$	failure criteria function
$T_0(x, y)$	temperature variation over the board;
$T_1(x, y)$	temperature variation through the thickness;
T_c	constant terms of $T_0(x, y)$
$T^0_{mn}, T^1_{mn}, T^c_{mn}$	Fourier series coefficients for $T_0(x, y)$, $T_1(x, y)$ and $T_c(x, y)$, respectively
r^*	stiffness coefficient
p^*	stiffness coefficient
s^*	stiffness coefficient
$u(x, y)$	in-plane deformation of laminate in the x direction
$v(x, y)$	in-plane deformation of laminate in the y direction
$w(x, y, t)$	transverse deflection
$\|W(t)\|$	transverse deflection magnitude
$(\sigma)^{(k)}$	kth lamina plane stress vector
σ^Y	yield strength of the lamina
τ^{F}	shear failure strength of the lamina
σ_i	stress in i direction, $i = x, y, xy$
σ^2_i	square of σ_i
ς	coefficient for thermal force N^{T}
ξ	coefficient for thermal moment M^{T}
ω	excitation frequency
ω_{mn}	linear system frequency
ω^{T}_{mn}	non-linear system frequency
PWB	printed wiring board

Non-linear dynamics theory of a thin laminate subject to mechanical loading has been developed by Whitney and Leissa [1]. Extensive studies have been made on the non-linear vibration and buckling response of the laminates [2]. Using non-linear plate theory, Suhir studied a printed wiring board (PWB) [3] in a single mode analysis for the plate deflection induced by a constant acceleration. The governing equation of motion for the non-linear deflection with von Karman strain field is reduced to a Duffing equation. An analytical solution is found to the Duffing equation subject to a constant loading in Suhir's analysis. The non-linear analysis of a laminate or plate dynamics in a multi-mode approach, however, often lead to a coupled modal form equation, with integrals of coupled modal functions in Fourier series resulting from reduction of the governing equation of motion by using the Galerkin's integral approach or energy method. The modal coupling is due to the non-linear coupling of the strain field equation and the governing equation of motion.