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International Journal of Non-Linear Mechanics 41 (2006) 57–77
INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS
www.elsevier.com/locate/nlm

Geometrically non-linear transverse vibrations of C–S–S–S and C–S–C–S[1] rectangular plates

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Accepted 1 April 2005

Abstract

This paper is concerned with a new improved formulation of the theoretical model previously developed by Benamar et al. based on Hamilton's principle and spectral analysis, for the geometrically non-linear vibrations of thin structures. The problem is reduced to a non-linear algebraic system, the solution of which leads to determination of the amplitude-dependent fundamental non-linear mode shapes, the frequency parameters, and the non-linear stress distributions. The cases of C–S–C–S and C–S–S–S rectangular plates are examined, and the results obtained are in a good qualitative and quantitative agreement with the previous available works, based on various methods. In order to obtain explicit analytical solutions for the first non-linear mode shapes of C–S–C–S RP[2] and C–S–S–S RP, which are expected to be very useful in engineering applications and in further analytical developments, the improved version of the semi-analytical model developed by El Kadiri et al. For beams and fully clamped rectangular plates, has been slightly modified, and adapted to the above cases, leading to explicit expressions for the higher basic function contributions, which are shown to be in a good agreement with the iterative solutions, for maximum non-dimensional vibration amplitude values up to 0.75 and 0.6 for the first non-linear mode shapes of C–S–C–S RP and C–S–S–S RP, respectively.

Keywords: Non-linear vibration; Hamilton's principle; Spectral analysis; Iterative procedure; Explicit formulation; Displacement basis; Modal basis

[1] The abbreviations C and S used along this paper for the plate boundary conditions correspond to clamped and simply supported edges, respectively.
[2] RP is an abbreviation for rectangular plates.
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0020-7462/$ - see front matter
doi:10.1016/j.ijnonlinmec.2005.06.002

1. Introduction

The subject of geometrically non-linear vibration of structures is of continuing interest, due to wide use of new materials, having a more accentuated non-linear behaviour, and the tendency to build more preferment structures, with high strength, high stiffness, and low weight, such as composites. It is then necessary to develop new design concepts, taking into account the non-linear behaviour induced by large vibration amplitudes, which may occur for example when structures working in a severe environment, are subjected to high acoustic excitation levels.

As the partial differential equation governing the transverse vibrations of thin elastic plates has no general analytical solution, no exact solutions are known, even in the linear case, for most of the rectangular plate boundary condition combinations. Consider the case of a square plate; 21 combinations of classical boundary conditions, i.e. clamped, simply supported, and free, exist and exact solutions are known only for the six cases having two opposite edges simply supported. For rectangular plates, due to the aspect ratio effect, the number of cases to be considered is much greater than 21. Furthermore, in a survey made by Leissa in 1973 [1], it was pointed out that until 1954, when Warburton derived his formulae based on a single-term representation of the deflection shapes for the natural frequencies of plates with various boundary conditions, no solution, even approximate, was known for six boundary condition cases. The general accuracy of Warburton's formulae is discussed in Ref. [1].

Generally, the plates are assumed, in structural dynamics theory, to be fully or partially free, simply supported, or clamped. The clamped support conditions assume that both displacements and rotations are prevented. This is difficult to achieve in practice [2]. However, these boundary conditions can be the most adequate for idealising practical structures, such as aircraft wing panels [3]. Although in such a situation, the real plate boundaries are neither perfectly clamped nor simply supported, due to the relative support flexibility, the natural frequencies and the stresses calculated on the basis of the fully clamped boundaries assumption are higher than those obtained in the simply supported boundaries case, and hence they may be considered by the designer as an upper limit. It appears from this example that it may be very useful for designers to be provided with results, practically easy to deal with, corresponding to various edge conditions. A comprehensive treatment of the linear problem and references corresponding to various combinations of edge conditions are given in the monograph of Leissa [4], and in his more recent review [1].

In the non-linear case, there have been in the last few decades a large number of studies dealing with many aspects of plate vibrations. Each problem has received a special treatment involving some particular approximations. The models proposed on the perturbation procedure, such as those proposed in [5,6] are practically limited to the first-order effects of finite displacements upon natural frequency. Also, in most of the studies carried out on non-linear vibrations of rectangular plates, the common approach to such problems has been to assume a spatial function, usually the linear mode shape, and seek a solution to the non-linear differential equation in time, which is generally of the Duffing type, using various solution techniques. In another series of works, the finite element method was extended to the non-linear case using different formulations, such as the hierarchical finite element method [7], the asymptotic numerical method [8], etc. However, in spite of the variety of approaches and solutions developed in the literature, simple practical analytical formulae, which may be easily implemented in design procedures, have to be extracted from the huge amount of the theoretical works available. A recent series of works have been devoted to this purpose and have led to simple analytical expressions for the non-linear mode shapes and resonant frequencies of various thin straight structures, such as beams, rectangular and circular plates. A theoretical model based on Hamilton's principle and spectral analysis was used in these works. It was developed in a general form and used for determination of the first three non-linear mode shapes of a clamped–clamped beam, the first and the second non-linear mode shapes of a fully clamped rectangular isotropic plate, the first non-linear mode shape of a fully clamped rectangular laminated plate, and the first and the second coupled circumferential mode shapes of isotropic circular cylindrical shells of infinite length [9–13]. It reduces the large vibration amplitude problem to a set of non-linear algebraic equations which is solved numerically for each value of the amplitude of vibration. An extra