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Pure global buckling and vibration in laminates with arbitrary shape cut off regions

C.C. Hong[a,*], H.W. Liao[b], M.F. Hwang[c], K.C. Jane[d]

[a]*Department of Mechanical Engineering, Hsiuping Institute of Technology, Taichung, 412 Taiwan, ROC*
[b]*Department of Information Technology, Ling Tung University, Taichung, 408 Taiwan, ROC*
[c]*Liberal Art Center, Da Yeh University, Changhua, 515 Taiwan, ROC*
[d]*Department of Applied Mathematics, National Chung Hsing University, Taichung, 402 Taiwan, ROC*

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Abstract

The pure global buckling and vibration of four sides simply supported as well as clamped anisotropic laminates having an arbitrary shape cut off region that is symmetric with respect to mid-plane have been studied by treating the remaining cut off regions as uniform plates with reduced stiffness. The reduced variation of stiffness of the plate is represented by Fourier series. Computational solutions of the energy principle for the Ritz method in a plate having arbitrary shape of typical cut off regions under biaxial compressive loads are obtained. Some numerical results for the pure global buckling load prediction due to its reduced flexural stiffness for the circular cut off regions and elliptical cut off regions are presented. We find the normalized pure global buckling load ratio decreases as the cut off regions size increases, and the non-dimensional fundamental frequency value decreases as the cut off regions size increases.

Keywords: Pure global buckling; Cut off region; Reduced stiffness; Arbitrary shape

1. Introduction

Cut off regions on the top and bottom surfaces of laminated plate may be necessary for fitting some equipment on it. Cut off regions may reduce bending stiffness of laminates, which lower the compressive load carrying capacity and natural frequencies. In 2003, Coffey present the excess dc conductivity with spectral cut off in closed form solution for buck anisotropic superconductors [1]. In 1985, Simitses et al. made the presents of delaminated homogenous laminate in axially load [2]. There are three possible buckling mode types: (a) local buckling mode, (b) global buckling mode and (c) coupled global and local buckling modes that have been examined. In 2000, Jane and Hong [3] made a study about the pure global buckling and vibration of rectangular laminates with rectangular cut off regions.

In this paper, an energy approach of the Ritz procedure is used to determine the pure global buckling load and vibration of four edges simply supported as well as clamped anisotropic rectangular laminates that having arbitrary shape cut off regions under biaxial compression loads. The reduced stiffness variation of the plate is represented by Fourier series and the partition technique [5] can be utilized to approach the arbitrary shape of cut off regions into the rectangular sub-regions and right triangular sub-regions. The purpose of this study is to investigate the effect of arbitrary shape cut off regions to the pure global buckling and vibration of rectangular plates by energy method. The typical anisotropic rectangular laminates with middle-plane symmetric arbitrary shape cut off region that is shown in Fig. 1. With coordinates $X_1(x_1, y_1)$, $X_2(x_3, y_1)$, $X_3(x_3, y_4)$, $X_4(x_2, y_3)$ and $X_5(x_1, y_2)$ and is used to demonstrate the study procedures. We partition the arbitrary shape of cut off region into the rectangular sub-regions and right triangular sub-regions as shown in Fig. 2. Where $\xi_{(k)}$ is the x-coordinate at middle point of $2c_{(k)}$, $2c_{(k)}$, which is the x-directional length of cut

*Corresponding author.
E-mail address: cchong@mail.hit.edu.tw (C.C. Hong).

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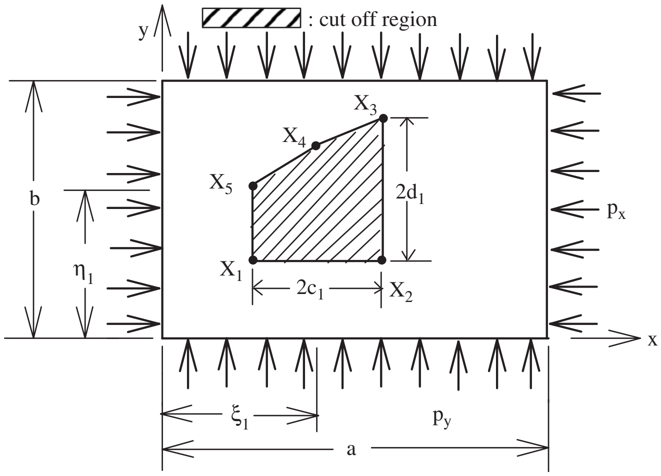

Fig. 1. Geometry of a typical laminated plate with arbitrary shape of cut off region.

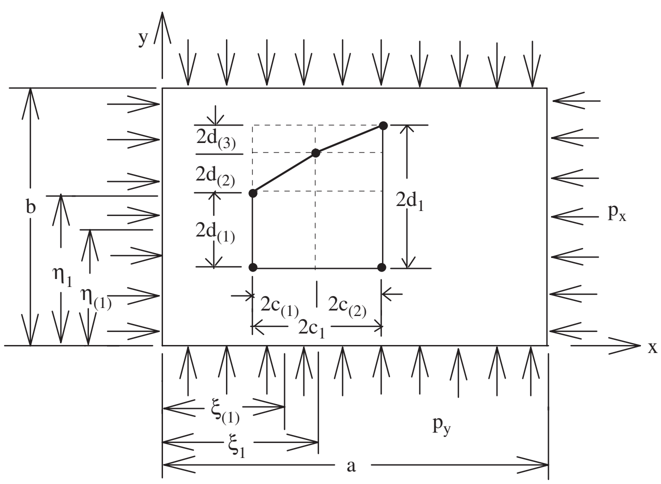

Fig. 2. Sub-regions of approximately arbitrary shape cut off region.

off sub-region and $\eta_{(l)}$ is the y-coordinate at middle point of $2d_{(l)}$, $2d_{(l)}$, which is the y-directional length of cut off sub-region.

2. Governing equations

2.1. Reduced flexural stiffness

When the arbitrary shape of cut off region is occurred in the rectangular plate, the overall effective flexural stiffness would be smaller than the flexural stiffness of perfect plate. We would like to partition the arbitrary shape of cut off region into sufficient numbers of rectangular shape of sub-regions and right triangular shape of sub-regions to get the approximate solution. A double Fourier series form of reduced flexural stiffness $\{\overline{D}\} = \{D\}f(x,y)$ is used in the laminated plate by Wang et al. [6]. Where $\{D\}$ representing original uniform homogeneous stiffness D_{11}, D_{12}, D_{66}, D_{16}, D_{26} and D_{66} and the distribution function of reduced flexural stiffness is written in the following form:

$$f(x,y) = a_{00} + \sum_{i=1}^{\infty} a_{i0} \cos \alpha_i x + \sum_{j=1}^{\infty} a_{0j} \cos \beta_j y + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} \cos \alpha_i x \cos \beta_j y, \qquad (1)$$

where a_{00}, a_{i0}, a_{0j} and a_{ij} are the Fourier coefficients, they are written in the following forms for the arbitrary shape of approximately cut off region that is shown in Fig. 2:

$$a_{00} = 1 - \frac{1}{A}\sum_{kl}^{KL} A_{kl}(1 - R_{kl}) - \frac{1}{A}\sum_{kl}^{TKL} TA_{kl}(1 - R_{kl})$$

$$a_{i0} = \frac{2}{a}\sum_{k=0}^{1}\int_{\xi_k + c_k}^{\xi_{k+1} - c_{k+1}} \cos \alpha_i x \, dx + \frac{2}{a}\sum_{k=1}^{K}\int_{\xi_{(k)} - c_{(k)}}^{\xi_{(k)} + c_{(k)}} \times \left[1 - \frac{2}{b}\sum_{l=1}^{L_k} d_{(l)}(1 - R_{kl})\right] \cos \alpha_i x \, dx + \frac{2}{a}\sum_{k}^{TK}\int_{\xi_{(k)} - c_{(k)}}^{\xi_{(k)} + c_{(k)}} \times \left[-\frac{1}{b}\sum_{l=1}^{TL_k} \frac{d_{(l)}}{c_{(k)}}(x - \xi_{(k)} + c_{(k)})(1 - R_{kl})\right] \cos \alpha_i x \, dx$$

$$a_{oj} = \frac{2}{b}\sum_{l=0}^{1}\int_{\eta_l + d_l}^{\eta_{l+1} - d_{l+1}} \cos \beta_j y \, dy + \frac{2}{b}\sum_{l=1}^{L}\int_{\eta_{(l)} - d_{(l)}}^{\eta_{(l)} + d_{(l)}} \left[1 - \frac{2}{a}\sum_{k=1}^{K_l} c_{(k)}(1 - R_{kl})\right] \cos \beta_j y \, dy + \frac{2}{b}\sum_{l}^{TL}\int_{\eta_{(l)} - d_{(l)}}^{\eta_{(l)} + d_{(l)}} \left[\frac{1}{a}\sum_{k=1}^{TK_l} \frac{c_{(k)}}{d_{(l)}}(y - \eta_{(l)} - d_{(l)})(1 - R_{kl})\right] \times \cos \beta_j y \, dy$$

$$a_{ij} = \frac{4}{ab}\int_0^b \sum_{k=0}^{1}\int_{\xi_k + c_k}^{\xi_{k+1} - c_{k+1}} \cos \alpha_i x \cos \beta_j y \, dx \, dy + \frac{4}{ab}\sum_{k=1}^{K}\left(\int_{\xi_{(k)} - c_{(k)}}^{\xi_{(k)} + c_{(k)}} \sum_{l=0}^{1}\int_{\eta_l + d_l}^{\eta_{l+1} - d_{l+1}} \cos \alpha_i x \cos \beta_j y \, dx \, dy + \int_{\xi_{(k)} - c_{(k)}}^{\xi_{(k)} + c_{(k)}} \sum_{l=1}^{L_k}\int_{\eta_{(l)} - d_{(l)}}^{\eta_{(l)} + d_{(l)}} R_{kl} \cos \alpha_i x \cos \beta_j y \, dx \, dy\right) + \frac{4}{ab}\sum_{k=1}^{TK}\left\{\int_{\xi_{(k)} - c_{(k)}}^{\xi_{(k)} + c_{(k)}} \sum_{l}^{TL_k}\int_{\eta_{(l)} - d_{(l)}}^{\eta_{(l)} + d_{(l)}} \left[1 + \frac{c_{(k)}}{d_{(l)}}(y - \eta_{(l)} - d_{(l)})(1 - R_{kl})\right] \cos \alpha_i x \cos \beta_j y \, dx \, dy\right\},$$

where $\alpha_i = i\pi/a$, $\beta_j = j\pi/b$, A_{kl} is the area of the klth cut off rectangular sub-region of $2c_{(k)} \times 2d_{(l)}$ dimensions with its center at $(\xi_{(k)}, \eta_{(l)})$ as shown in Fig. 2, KL is the total number of cut off rectangular sub-regions, $A = ab$, R_{kl} is the ratio of the amount of flexural stiffness, same for all components, for the kl cut off sub-region to the corresponding stiffness of the no cut of laminate, $\xi_0 = c_0 = 0$, $\xi_2 = a$, $c_2 = 0$, $\eta_0 = d_0 = 0$, $\eta_2 = b$, $d_2 = 0$, K is the total number of cut off rectangular sub-regions in the