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Uncertainty of residual stresses measurement by layer removal

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Abstract

A model to evaluate the uncertainty in the measurement of the through-thickness residual stress distribution in plates by the layer removal technique is presented. Thin layers were chemically etched from a stripe on rectangular specimens cut from a low carbon cold-rolled steel sheet. Phase shifting laser interferometry was used to measure the ensuing curvature. Polynomials were least-squares adjusted to the curvatures as a function of the etched depth. The polynomials were inserted into an integro-differential equation relating the curvature to the residual stresses, which were assumed to be a function of depth only. A comparison with X-ray diffraction measurement of the surface residual stresses showed good agreement. The uncertainty was found to increase steeply at the surfaces and to depend mainly on the assumed value for the modulus of elasticity, on the curvature fit, and on the depth of etching.

Keywords: Residual stresses; Layer removal; Uncertainty; ESPI

1. Introduction

Residual stresses arise commonly in many forming or processing operations, especially in those that involve large applied forces or severe temperature gradients. They may affect the lifetime of manufactured products by fatigue or environmentally induced degradation, alter the performance of these products by crazing or changes in shape, bring about their failure by brittle fracture, and extend or reduce their load carrying capacity.

Since the presence of residual stresses may have significant consequences, their measurement is clearly important for improved process and product control, design, performance and modeling. Various measurement techniques have been developed; these techniques are either non-destructive or locally destructive. The former include X-ray diffraction, neutron diffraction and magnetic or ultrasonic methods; the latter include mechanical means such as hole drilling and layer removal methods [1,2]. However, since in all cases experimental errors are inevitable, there will exist always an uncertainty associated with the measurement results.

Although some papers related to the evaluation of the uncertainty in the measurement of residual stresses have been published [3–14], it appears that most of them do not conform to internationally accepted recommendations [15]. The embryo of a metrological system for residual stress analysis has been proposed [16], by which it is hoped that the accuracy of X-ray diffraction measurements can be properly assessed in the future.

As a further contribution to this subject, in this paper we propose a methodology to analyze the uncertainty of the results obtained by the layer removal. The technique is based on the fact that residual stresses within the material are in a state of self-equilibrium; after part of the material is taken away, the stress distribution changes and the sample bends to restore balance. For example, removal of a surface layer of a plate with a longitudinal stress distribution generates a bending moment. By measuring the curvature after eliminating successive layers through the thickness, the initial distribution can be reconstructed [1,17]. It is possible to measure directly the radius of curvature with a laser sensor [6]; to measure the deflection

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Nomenclature

a	original specimen thickness
C	curvature
c	sensitivity coefficient
$\mathbf{D}$	derivative matrix
d	depth
E	Young's modulus
e	error
$\mathbf{F}$	function matrix
f	functional relation
I	light intensity, number of points
l	distance in pixels
M	optical magnification
N	number of fringes
p	polynomial coefficient
r	distance between fringes
s	standard deviation
u	standard uncertainty
$\mathbf{U}$, $\mathbf{u}^2$	uncertainty matrix
w	width
x, y, z	Cartesian coordinates

Greek symbols

α, β	angles of incident and scattered beams
δ	width of rectangular probability density function
ϕ	bending angle
φ	increment of bending angle
σ	residual stress
ν	Poisson's ratio
λ	laser wavelength

with a mechanical or electromechanical dial [17]; or to measure the curvature indirectly as in [18], which reference the present article follows closely.

We used chemical etching to remove progressively thin layers from 1 mm thick, cold-rolled, low carbon steel rectangular specimens cut from the same blank. The specimens were mounted as cantilever beams and the acid attack was restricted to a narrow transversal stripe. The curvature was determined by means of temporal phase shifting electronic speckle pattern interferometry (ESPI) [19,20]. Through-thickness measurements were performed by using two specimens that were machined from opposite sides [6]. The curvatures were fitted to polynomials whose coefficients took into account the uncertainty in both coordinates [21–23]. These polynomials were then inserted into the model developed by Treuting and Read [24] and the law of propagation of uncertainty [15] was applied. Computed residual stresses at the surface were found to agree with standard X-ray measurements performed before machining the samples. The uncertainty was found to increase steeply at the surfaces and to depend mainly on the assumed value for Young's modulus, on the fit to the curvature, and on the depth of etching.

2. Experimental procedure and results

2.1. Material and assumptions

Two rectangular $100\,\text{mm} \times 25\,\text{mm} \times 1\,\text{mm}$ specimens were cut far from the borders of a commercial low carbon, cold-rolled steel sheet. Very low speed machining was used to avoid introducing extraneous residual stresses at the edges of the specimens. It was assumed that the principal stresses coincided with the rolling (x) and transverse (y) directions.

The specimens were clamped as cantilever beams. One of their surfaces was covered with plastic tape, except for a transversal stripe of width $w = 3\,\text{mm}$ in the y direction. One edge of the stripe was 25 mm away from the clamped end, this left about 52 mm from the other edge of the stripe to the free end, see Fig. 1.

Nitric acid at 45% concentration in volume was carefully applied in steps over the stripe in order to remove successive layers. Stress readjustment then forced the specimens to bend. The $x-y$ plane was kept vertical to facilitate the flow of acid and to minimize curvature changes due to diminishing specimen weight. After the ith removal, the depth d_i of the groove and the small angle ϕ_i between the free end of the specimen and its original location were measured. This allowed calculating the curvature of the stripe as

$$C_i = \frac{\tan\,\phi_i}{w}. \tag{1}$$

A polynomial was then adjusted to the (d_i, C_i) set of points in order to produce a continuous curvature function $C(z)$. Finally, the residual stress at different values Z of the thickness coordinate z was computed from the measurement model [24]

$$\sigma_x(Z) = \frac{E}{6(1-\nu^2)}\left[(a-Z)^2 \frac{\mathrm{d}C}{\mathrm{d}z}\bigg|_{z=Z} + 4(a-Z)C(Z) - 2\int_0^Z C(z)\,\mathrm{d}z\right], \tag{2}$$

where a is the thickness of the specimen, E is Young's modulus and ν is Poisson's ratio. In Eq. (2), the origin $z = 0$ is at the surface of the specimen.

This model assumes that the geometry of groove is ideal, as depicted in Fig. 1; that the material properties are homogeneous; that the layer removal process does not introduce additional stresses and that the stresses of the remaining material keep within the elastic range. To comply with the fourth requirement, the maximum depth of the groove was determined from the formula in