Journal of Molecular Liquids 264 (2018) 375–385

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Entropy generation for flow of Sisko fluid due to rotating disk

Tasawar Hayat[a,b], Muhammad Ijaz Khan[a,*], Sumaira Qayyum[a], Muhammad Imran Khan[c,*], A. Alsaedi[b]

[a] Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
[b] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O.Box 80257, Jeddah 21589, Saudi Arabia
[c] Heriot Watt University, Edinburgh Campus, Edinburgh EH14 4AS, United Kingdom

ARTICLE INFO

Article history:
Received 27 March 2018
Received in revised form 23 April 2018
Accepted 5 May 2018
Available online xxxx

Keywords:
Sisko fluid
Nonlinear thermal radiation
Viscous dissipation
Entropy generation
Joule heating
Rotating stretchable disk

ABSTRACT

This investigation deals with entropy generation optimization and nonlinear dissipative flow of Sisko fluid due to a rotating disk. Joule heating effect is considered. Mathematical model is obtained for the present flow problem. Nonlinear systems are solved for convergent series solutions. Analysis of various flow variables on Bejan number, volumetric entropy generation rate, temperature and velocity are discussed. Present analysis depicts that irreversibility rate (entropy generation rate) and Bejan number are enhanced for radiation parameter, whereas reverse behavior is noticed for Brinkman number and material parameter. The velocity and temperature fields are increasing for larger material parameter and temperature ratio parameter. The obtained results are helpful in understanding the irreversibility (entropy generation) for flow of non-Newtonian fluids. Comparative study is made for temperature, velocity, Bejan number and entropy generation by considering shear thickening and thinning fluids. Finally, a comparison is given with the previous available results.

1. Introduction

Flows due to rotating disk have gained considerable attention in numerous engineering and industrial processes. Rotating disk has utilizations in different industries, for example lubrication, rotating machinery, computer storage devices and oceanography etc. To best of our information, Von Karman [1] initially investigated liquid flow by an infinite disk. In this study he introduced famous similarity transformations. Hayat et al. [2] recently studied dissipative flow of second grade liquid by rotating disk with Joule heating. Homotopy analysis technique is implemented to construct the series solution of nonlinear ordinary equations. Nanomaterial (nanofluid) under investigation depends upon thermophoresis and Brownian moment. Their obtained outcome indicates that velocity of fluid particles increases through rising values of viscoelastic parameter. Furthermore velocity gradient has contrast behavior for higher magnetic and viscoelastic variable, while Nusselt number is higher for larger Reynolds number. Yao and Lian [3] presented flow due to rotating. In this investigation both analytical and numerical solutions are obtained when the fluid is in state of rigid body rotation. Analytical solution of considered flow is meaningful when $0 \leq s \leq 1$. For $-0.1575 \leq s \leq 0$, Runge-Kutta technique along with Newton-Raphson technique is implemented to obtain the first solution branch of dual solutions. Mahanthesh et al. [4] scrutinized nonlinear radiative nanomaterial flow by a rotating disk. The considered nanomaterial flow is examined in the presence of Titanium Alloy (Ti_6Al_6V) nanoparticles. Furthermore lamina, tetrahedron, sphere, column and hexahedron shapes of nanoparticles are considered in this study. Heat flux condition instead of constant surface temperature is used. Runge-Kutta-Fehlberg method is implemented to obtain the computational results of governing equations. Ellahi et al. [5] discussed ferrofluid flow due to stretchable rotating disk. Von Karman transformations are used to covert the PDEs into coupled ordinary differential equations. Convergent series solutions by HAM (homotopy analysis method) is given. Both velocity and temperature strongly depend upon effective magnetization variable. Velocity decreased by higher effective magnetization variable while reverse behavior is observed for such estimation on temperature.

The non-Newtonian fluids are now much suited in industry and technological processes. Such fluids for diverse properties are many in nature. Thus various liquid models of non-Newtonian materials have been suggested. Sisko fluid is one of such fluid model. Polymeric suspensions, for example waterborne coating are Newtonian

* Corresponding authors.
E-mail addresses: mikhan@math.qau.edu.pk (M.I. Khan), mk42@hw.ac.uk (M.I. Khan).

https://doi.org/10.1016/j.molliq.2018.05.022
0167-7322/© 2018 Published by Elsevier B.V.

and follow the Sisko model. The viscosity of such material strongly depends on shear rate and strain. Very little information about Sisko fluid model [6] is accessible in literature. Tanveer et al. [7] studied peristaltic flow of Sisko material in curved channel. Ijaz et al. [8] worked on entropy generation optimization in flow of Sisko liquid. Heat generation/absorption and radiation are taken. Khan et al. [9] examined double diffusion effects in flow of Sisko liquid. Bhatti et al. [10] examined peristaltic flow of non-Newtonian fluid with titanium-magneto nanomaterials.

Heat transport through nonlinear radiative heat flux has numerous applications for heat exchangers, electronic cooling etc. Waleed et al. [11] scrutinized nonlinear radiative nanomaterial flow and entropy generation minimization by a thin needle. Kumar et al. [12] studied chemical reaction and nonlinear thermal radiation effects in nanomaterial flow by a stretched surface. Khan et al. [13] examined magneto-Burgers nanomaterial flow with gyrotactic microorganisms and nonlinear radiative heat flux. Babu and Sandeep [14] presented bioconvective stagnation point flow towards a stretched sheet. Prasannakumara et al. [15] worked on nonlinear radiative flow of Sisko nanomaterial over a nonlinear stretched surface. Few recent studies about entropy generation can be seen via Refs. [16-20].

The purpose of present article is to investigate irreversibility of nonlinear radiative flow of Sisko fluid by rotating disk. MHD fluid is taken. Joule heating and dissipation are considered. Total entropy generation rate through different variables are calculated. Convergent series solutions by homotopy analysis method [21-30] are obtained. Entropy generation rate and Bejan number are discussed. Comparison between the present and previous results is seen.

2. Mathematical description and coordinate system

Main theme of our analysis is to investigate the entropy generation in non-Newtonian (Sisko fluid model) fluid flow due to rotating stretchable disk. Heat transfer is taken with Joule heating, viscous dissipation and nonlinear thermal radiation. Disk at $z=0$ has stretching rate (a_1) and angular velocity (Ω_1) respectively. Magnetic field of strength B_0 is applied in flow direction (z−direction). Here disk and ambient temperatures are denoted by $\hat{T}_w$ and $\hat{T}_\infty$ while disk and ambient concentration are defined by $\hat{C}_w$ and $\hat{C}_\infty$ respectively (see Fig. 1). The conservation laws of mass, momentum and energy yield [6,7]:

$$\frac{\hat{u}}{r}+\frac{\partial\hat{u}}{\partial r}+\frac{\partial\hat{w}}{\partial z}=0, \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial r}-\frac{\hat{v}^2}{r}+\hat{w}\frac{\partial\hat{u}}{\partial z}\right)=-\frac{\partial\hat{p}}{\partial r}+\frac{1}{r}\frac{\partial(r\bar{S}_{rr})}{\partial r}+\frac{\partial(\bar{S}_{zr})}{\partial z}-\frac{\bar{S}_{\psi\psi}}{r}-\sigma B_0^2\hat{u}, \tag{2}$$

$$\rho\left(\hat{u}\frac{\partial\hat{v}}{\partial r}+\frac{\hat{u}\hat{v}}{r}+\hat{w}\frac{\partial\hat{v}}{\partial z}\right)=-\frac{1}{r}\frac{\partial\hat{p}}{\partial\psi}+\frac{1}{r^2}\frac{\partial(r^2\bar{S}_{r\psi})}{\partial r}+\frac{\partial(\bar{S}_{z\psi})}{\partial z}+\frac{\bar{S}_{\psi r}-\bar{S}_{r\psi}}{r}-\sigma B_0^2\hat{v}, \tag{3}$$

$$\rho\left(\hat{u}\frac{\partial\hat{w}}{\partial r}+\hat{w}\frac{\partial\hat{w}}{\partial z}\right)=-\frac{\partial\hat{p}}{\partial z}+\frac{1}{r}\frac{\partial(r\bar{S}_{rz})}{\partial r}+\frac{\partial(\bar{S}_{zz})}{\partial z}, \tag{4}$$

$$(\rho c_p)\left(\hat{u}\frac{\partial\hat{T}}{\partial r}+\hat{w}\frac{\partial\hat{T}}{\partial z}\right)=k\left(\frac{\partial^2\hat{T}}{\partial z^2}+\frac{1}{r}\frac{\partial\hat{T}}{\partial r}+\frac{\partial^2\hat{T}}{\partial r^2}\right)-\nabla.q_r+\bar{S}:L+\sigma B_0^2(\hat{u}^2+\hat{v}^2), \tag{5}$$

$$\left.\begin{array}{c}
\bar{S}_{rr}=\left(2\frac{\partial\hat{u}}{\partial r}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),\ \ \bar{S}_{\psi\psi}=\left(2\frac{\hat{u}}{r}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),\\
\bar{S}_{zz}=\left(2\frac{\partial\hat{w}}{\partial z}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),\ \ \bar{S}_{r\psi}=\bar{S}_{\psi r}=\left(\frac{\partial\hat{v}}{\partial r}-\frac{\hat{v}}{r}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),\\
\bar{S}_{zr}=\bar{S}_{rz}=\left(\frac{\partial\hat{u}}{\partial z}+\frac{\partial\hat{w}}{\partial r}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),\ \ \bar{S}_{z\psi}=\bar{S}_{\psi z}=\left(\frac{\partial\hat{v}}{\partial z}\right)\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right),
\end{array}\right\} \tag{6}$$

$$\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}=\left|\left(\frac{\partial\hat{u}}{\partial z}+\frac{\partial\hat{w}}{\partial r}\right)^2+2\left(\frac{\partial\hat{w}}{\partial z}\right)^2+\left(\frac{\partial\hat{v}}{\partial r}-\frac{\hat{v}}{r}\right)^2+2\left(\frac{\hat{u}}{r}\right)^2+\left(\frac{\partial\hat{v}}{\partial z}\right)^2+2\left(\frac{\partial\hat{u}}{\partial r}\right)^2\right|^{\frac{n-1}{2}}, \tag{7}$$

$$\bar{S}:L=\left[\left(\frac{\partial\hat{u}}{\partial z}+\frac{\partial\hat{w}}{\partial r}\right)^2+2\left(\frac{\partial\hat{w}}{\partial z}\right)^2+\left(\frac{\partial\hat{v}}{\partial r}-\frac{\hat{v}}{r}\right)^2+2\left(\frac{\hat{u}}{r}\right)^2+\left(\frac{\partial\hat{v}}{\partial z}\right)^2+2\left(\frac{\partial\hat{u}}{\partial r}\right)^2\right]\left(\alpha+\beta\left|\frac{1}{2}trA^{*2}\right|^{\frac{n-1}{2}}\right), \tag{8}$$

$$q_r=\frac{-16\sigma^*\hat{T}^3}{3k^*}\frac{\partial\hat{T}}{\partial z}. \tag{9}$$

Considering

$$\begin{aligned}
\tilde{u}&=\frac{\hat{u}}{R\Omega_1},\ \ \tilde{v}=\frac{\hat{v}}{R\Omega_1},\ \ \tilde{w}=\frac{\hat{w}}{\delta\Omega_1},\ \ \tilde{T}=\frac{\hat{T}-\hat{T}_\infty}{\hat{T}_\infty},\\
\tilde{r}&=\frac{r}{R},\ \ \tilde{z}=\frac{z}{\delta},\ \ \tilde{p}=\frac{\hat{p}}{\rho(R\Omega_1)^2},
\end{aligned} \tag{10}$$