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Notched shells with surface cracks under complex loading

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Abstract

In the present paper, a double-curvature thin-walled shell with a circular-arc notch is considered. For different notch configurations, the stress-intensity factor (SIF) along the front of an elliptical-arc surface crack at the notch root is computed through a three-dimensional finite element analysis, for seven elementary stress distributions perpendicular to the crack faces. Then, by employing such results, an approximate expression of SIF in the case of a generic complex loading is determined by means of the power series expansion of the actual stress field and the superposition principle. Finally, as an example of how to apply the above numerical procedure, the effect of the stress concentration on the SIF values is examined for both cylindrical and spherical notched shells under an internal pressure.

Keywords: Notched thin-walled shell; Surface crack; Stress-concentration factor (SCF); Stress-intensity factor (SIF)

1. Introduction

Stress concentration in a notched structural component can provoke the initiation and growth of a surface crack at the notch root [1,2]. As is well-known, the stress field near such a defect can be quite different from that determined in an unnotched component with an identical flaw.

Several authors have examined the influence of surface cracks in smooth structural components such as round bars [3–6], pipes and shells [7–10], whereas cracked round bars [11,12] and pipes [13–15] with hoop grooves have been considered in some papers. Only a few authors [16–18] have analysed the behaviour of part-through-cracked double-curvature shells.

Many structural components, such as pressure vessels, pipe elbows, fuel tanks and so on (Fig. 1), may be considered as double-curvature shells since they have two distinct principal curvature radii (at least in some portions). These components frequently present geometrical discontinuities (changes in cross-section sizes, fillets, etc.), and are usually obtained by assembling different parts jointed together by welding processes. Such geometrical variations, often modelled by means of semi-circular notches, are preferential sites for crack nucleation and propagation.

A portion of a notched double-curvature thin-walled shell is herein represented as a part of a shell of revolution. The notch profile is assumed to belong to one of the two planes defined by the principal curvature radii of the shell. A surface defect may initiate because of damage or stress concentration. Such a part-through flaw is assumed to be located at the notch root, to lie in one of the above two planes, and to present an elliptical-arc shape (Fig. 2). Further, the crack plane is perpendicular to the notch profile.

The dimensionless stress-intensity factors (SIFs) for seven different elementary opening stress distributions (constant, linear, quadratic, cubic, quartic, fifth and sixth order) acting on the crack faces are determined through a three-dimensional finite element analysis, by considering different notch configurations. Some results are compared with those by other authors.

Since the stress field highly depends on the loading conditions (for example, internal pressure, temperature gradients, residual stresses, fretting stresses and so on), a simple procedure to compute the SIFs for different loading types is herein proposed. More precisely, in the case of a generic complex loading, an approximate expression of SIF can be determined by employing: (1) the SIF values

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Nomenclature

- a — crack depth for the deepest point A on the crack front
- a, b — semi-axes of the ellipse
- $B_{i(L)}$ — ith coefficient of the power series expansion for a generic complex loading (case L)
- c — notch depth
- h — distance of point B from the Y-axis
- $K_{I(i)}$, $K^*_{I(i)}$ — stress-intensity factor (SIF) and dimensionless SIF, respectively, for the ith elementary opening stress $\sigma_{I(i)}$
- $K_{I(L)}$, $K^*_{I(L)}$ — stress-intensity factor (SIF) and dimensionless SIF, respectively, for the complex opening stress $\sigma_{I(L)}$
- $K^*_{I(p)}$ — dimensionless SIF for internal pressure
- $K_{t(p)}$ — stress-concentration factor (SCF) for internal pressure
- $r^* = R_1/R_2$ — relative curvature radius of the shell
- R_1, R_2 — principal curvature radii of the shell
- $R = \min(R_1 - t, R_2 - t)$ — internal radius of the shell
- t — wall thickness of the shell
- $t^* = t/R$ — dimensionless wall thickness of the shell
- $t' = t - c$ — reduced wall thickness of the shell in the notched zone
- u — radial coordinate (its origin O is on a circle with the centre in O_1 and radius $R_1 - c - a$; see Fig. 5)
- $\alpha = a/b$ — aspect ratio of the elliptical-arc crack front
- $\delta = c/t$ — relative notch depth
- $\gamma = u/a$ — dimensionless radial coordinate (its origin O is on a circle with the centre in O_1 and radius $R_1 - c - a$; Fig. 5)
- $\sigma_{I(i)}$ — stress perpendicular to the crack faces (Mode I, opening) for the ith elementary load case
- $\sigma_{I(L)}$ — opening stress for a generic complex loading (case L)
- $\sigma_{I(p)}$ — opening stress for internal pressure
- $\sigma_{ref(i)}$ — reference stress for the ith elementary load case
- $\sigma_{ref(L)}$ — reference stress for a generic complex loading (case L)
- $\sigma_{ref(p)}$ — reference stress for internal pressure
- ρ, $\rho_d = \rho/t$ — notch radius and dimensionless notch radius
- $\xi = a/\bar{t}$ — relative crack depth for point A on the crack front ($\bar{t}$ is equal to t or t' for an unnotched or a notched shell, respectively)
- $\zeta, \zeta^* = \zeta/h$ — coordinate and normalized coordinate, respectively, of the generic point P along the crack front

Subscripts

- c — cylindrical shell
- d — dimensionless
- $i = 0, \ldots, 6$ — index related to the generic elementary opening stress
- L — index related to a generic complex loading
- n — notched shell
- p — pressure
- s — spherical shell
- u — unnotched shell

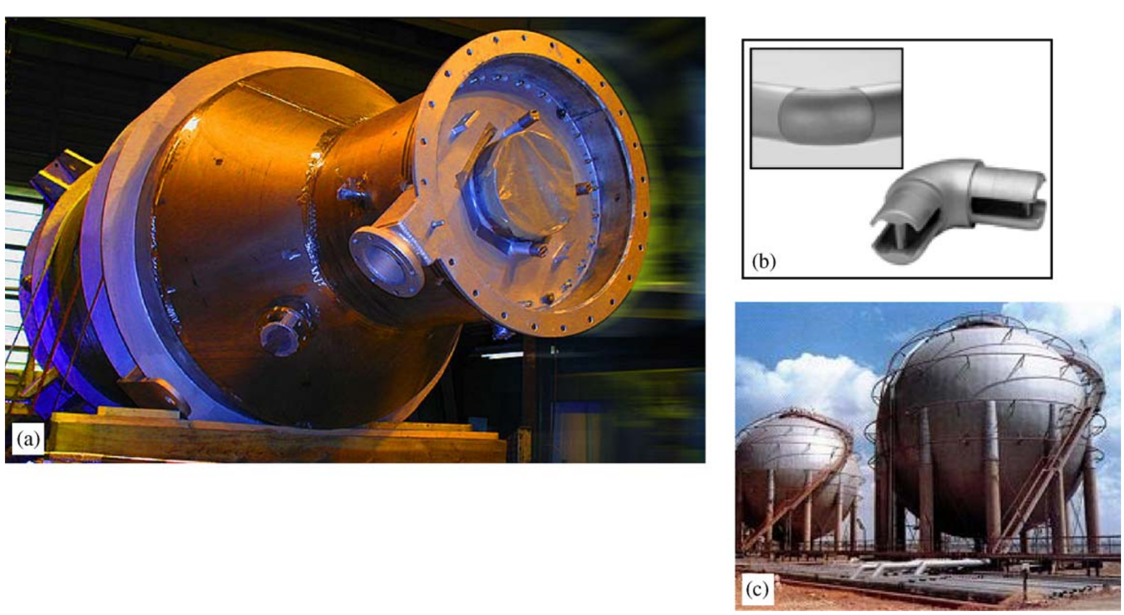

Fig. 1. Examples of structural components with a double-curvature shell shape: (a) pressure vessel; (b) pipe elbow and (c) spheres for liquid and gas storage.