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Determining the up-down-up response through tension tests of a pre-twisted shape memory alloy tube

Pingping Zhu [a], Ping Feng [b], Qing-Ping Sun [c], Jiong Wang [d], Hui-Hui Dai [a], *

[a] Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong
[b] College of Mechatronics and Control Engineering, Shenzhen University, Shenzhen, 518060, China
[c] Department of Mechanical Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong
[d] Department of Mechanics, South China University of Technology, 381 Wushan Road, Tianhe District, Guangzhou, 510641, China

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ABSTRACT

In this paper, the stress-induced phase transitions in a shape memory alloy (SMA) tube under torsion and pre-twisted tension are studied analytically. A constitutive model with the specific forms of Helmholtz free energy and mechanical dissipation rate is employed to formulate the governing system. Exact solution of the tube under pure torsion is first derived and the shear stress–shear strain response is determined, which reveals the hardening effect. For the pre-twisted tube under uniaxial tension, the one-dimensional asymptotic tensile stress–tensile strain relations for the austenite, the phase transition and the martensite regions are derived by using the asymptotic expansion method. By properly defining an elastic energy potential, the present system can be viewed as an elastic problem, which can be related to the problem of Ericksen's bar. The analytical formulas for the nucleation and propagation stresses in terms of the pre-shear strain (caused by the pre-twist) are obtained. Tension tests with fixed pre-twists on SMA thin-walled tube are conducted, with a focus on the stress–strain response. The measured values and the analytical formula for the propagation stress are used to determine the material parameters, which, in turn, yields the up-down-up response of a shape memory alloy tube under pure tension. The tendency and turning point of the phase transformation in the pre-twisted tube from localization to homogeneous deformation are also determined, which suggests a plausible way to avoid the instability in actuation applications.

1. Introduction

Shape memory alloys (SMAs) have been successfully applied in a variety of fields, including medicine, aerospace and automation (see Jani et al., 2014). The thermo-mechanical behaviors of this group of materials exhibit two remarkable properties: the shape memory effect (SME) and the pseudo-elasticity (PE). The SME means that at low temperatures SMAs can be deformed under external loads and the deformations can be recovered upon heating above the transition temperature. The PE means that at appropriate (high) temperatures, SMAs are able to recover their undeformed state if the applied external forces are removed. The underlying mechanisms of these two unusual properties are the stress- or temperature-induced phase transitions between the austenite (A) phase and the (twinned or detwinned) martensite (M) phase.

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* Corresponding author.
E-mail address: mahhdai@cityu.edu.hk (H.-H. Dai).

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In order to gain a deep understanding of the thermo-mechanical response of SMAs, a series of experiments, including the uniaxial and the biaxial loading tests, have been conducted on SMA specimens with different geometries. In Li and Sun (2002), He and Sun (2009), Zhou and Sun (2011), the behaviors of SMA thin-walled tubes under various loading conditions have been systematically investigated. In the pure tension tests, it was found that the deformation of the tubes can be inhomogeneous, which is macroscopically reflected in the nucleation and propagation of martensite/austenite bands with sharp A-M interfaces. The stress–strain curve during a whole tensile loading cycle forms a hysteresis loop, which implies that energy dissipation occurs during the phase transition process. On the other hand, the responses of SMA thin-walled tubes under pure torsion or combined tension–torsion loading conditions have also been systematically investigated (see Lim and McDowell, 1999; Sun and Li, 2002; McNaney et al., 2003; Mehrabi et al., 2012; Wang et al., 2012; Mehrabi and Kadkhodaei, 2013; Mehrabi et al., 2015a,b). In Sun and Li (2002), it was found that the deformation of the tube under pure torsion is axially homogeneous and the recorded stress–strain curves for both the loading and unloading processes represent monotonic hardening. In the tension tests of a pre-twisted SMA tube, it was found that the deformation of the tube changes gradually from the localization form to the homogeneous form with the level of the pre-twisted angle gradually increasing. The recorded tensile stress–tensile strain curve becomes monotonically hardened. The von-Mises circle plotted in Sun and Li (2002) and the relevant surface morphology observation showed that the tube has a strong anisotropic response during stress-induced transformations. Significant differences of the responses of SMA specimens in torsion and tension have also been reported in Wang et al. (2012), Mehrabi and Kadkhodaei (2013), Mehrabi et al. (2015a,b). Among these works, Mehrabi et al. (2015b) investigated the responses of NiTi thin-walled tubes under proportional and non-proportional tension–torsion tests based on stress-control or strain-control loading patterns. They also found the anisotropic response of the NiTi tubes in phase transitions.

For the instability phenomena of SMAs observed in the tension tests, the localization configuration of the tube is unfavorable for some structural and actuation applications. Based on this consideration and the above mentioned experimental investigations, two issues are of interest in our present work.

First, studies have focused on finding the true material response over the Lüders-like stress plateau in literature (see Shaw and Kyriakides, 1997; Shaw and Kyriakides, 1998; Li and Sun, 2002). However, it is rather difficult to calibrate material parameters in many proposed constitutive models and to deduce the softening part of the stress–strain curve. Some works have subtly overcome this difficulty. For example, Hallai and Kyriakides (2013) produced the up-down-up stress–strain response of a NiTi strip by using an experimental technique that was first reported in Shioya and Shiroiri (1976). They conducted uniaxial tension tests on a laminate (consisting of two stainless steel strips as the face-strips and one NiTi layer as the core) and the stainless steel, respectively. The stainless steel is known to be a hardening material while NiTi is an unstable material. Under proper design, the instability of the core in such laminates was found to be suppressed by the hardening effect of the face-strips. This was structurally reflected in the recorded monotonic hardening stress–strain curves of the laminate. Subsequently, they extracted the material response of NiTi from the response of the laminate and the face-strips, which was found to have an up-down-up fashion. Moreover, Song et al. (2013) proposed an analytical approach based on the constitutive model proposed in Rajagopal and Srinivasa (1999). They adopted specific forms of the Helmholtz free energy and the rate of mechanical dissipation to study phase transformations of a NiTi wire under uniaxial tension. By taking advantage of the small strains of SMAs, they used the asymptotic expansion method developed in some previous works (see Cai and Dai, 2006; Dai and Cai, 2006; Dai and Wang, 2009, 2010; Wang and Dai, 2010; Wang and Dai, 2012a,b) to tackle the problem analytically. By further relating the problem to the equilibrium theory of Ericksen (1975), they deduced the analytical formulas of the nucleation and propagation stresses, which were used to calibrate material constants by comparing with the relevant experimental data. Consequently, the nominal stress–strain curve was captured, which also shows an up-down-up fashion. One deficiency of this asymptotic analytical study is that determining the material constants requires nucleation stresses. Generally speaking, the value of the nucleation stress is difficult to be measured in the experiments. On the other hand, the propagation stress can be clearly read out from the measured stress–strain curves. In this regard, we aim at providing another analytical approach to capture the material response of SMAs by using the propagation stress measured in the experiments.

More specifically, we study the stress-induced phase transitions in an SMA thin-walled tube based on a three-dimensional setting. Homogeneous and piecewise homogeneous deformations of such a tube under pre-twisted tension are considered. We aim at obtaining the analytical formulas of the nucleation and propagation stresses in terms of the material constants and the pre-shear strain (induced by the pre-twist). In order to determine the material constants, a series of tension tests with fixed pre-twists on SMA thin-walled tube are conducted, with a focus on the stress–strain response. The measured values and the analytical formula for the propagation stress are then used to determine the material constants. Subsequently, an up-down-up tensile stress–tensile strain curve is captured.

Another motivation of the current work comes from some previous efforts made to widen the controllable range of SMAs by creating transformation loading gradient along the deforming direction (see Mahmud et al., 2007; Mahmud et al., 2008; Shariat et al., 2012, 2013). Geometrically graded and microstructurally graded NiTi alloys are usually used to achieve this goal. Noticing the undesirable effects on some actuation applications the material instability of SMAs has, Shariat et al. (2012) tapered a NiTi bar to provide a gradually changing cross-sectional area so that the uniaxial tensile stress varies along the length of the bar. This treatment geometrically created a transformation stress gradient along the length of the bar,