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Pre-compensation of servo contour errors using a model predictive control framework

Shiyi Yang, Amir H. Ghasemi, Xiangxing Lu, Chinedum E. Okwudire*

Mechatronics and Sustainability Research Laboratory, Department of Mechanical Engineering, University of Michigan, 2350 Hayward, Ann Arbor, MI 48109, United States

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ABSTRACT

Methods for pre-compensating contour errors in servo systems by adding components of the predicted contour error to the reference position command have recently been proposed in the literature. Such methods are very effective when the curvatures of the desired path are small but their performance degrades at locations of sharp curvature because they lack look-ahead capabilities. This paper presents an improved method for pre-compensating contour errors in servo systems by modifying reference position commands using a model predictive control framework. The pre-compensation value at any given location along the desired path is defined as a weighted average of contour errors within a prediction horizon, and the weights are selected to minimize the sum of squares of the estimated contour errors over the chosen prediction horizon. Constraint enforcement functionalities are also built into the proposed method to ensure that the pre-compensated reference commands stay within specified velocity and acceleration limits. Simulations and experiments are used to compare the performance of the proposed method to a recently proposed pre-compensation approach which lacks look-ahead and constraint enforcement capabilities. Significant improvements in contouring accuracy over the existing method are demonstrated.

1. Introduction

The reduction of contour errors is a subject of great importance for CNC machines because contour errors directly affect the ability of the manufactured part to meet tolerance specifications [1–5]. Contour errors are generated when at least one of the machine axes is unable to perfectly follow the desired trajectory [6,7] due to the limited bandwidth of its servo controller, influence of external disturbances such as friction and cutting forces, and limitations in the actuators and sensors. This paper focuses on contour errors caused by the limited bandwidth of servo controllers.

Contour errors resulting from limited controller bandwidth can be reduced either by improving the controller to achieve better contouring control or by modifying the reference commands sent to the machine so as to pre-compensate for contour errors. Better contouring control is most often achieved indirectly by designing high-bandwidth controllers that help minimize axis tracking errors through feed forward and feedback axis-level control (e.g., [8–11]). However, reducing tracking errors does not necessarily reduce contour errors [12]. Therefore, Koren [1] introduced the cross-coupled controller (CCC) to help control contour errors directly. A typical CCC consists of an algorithm to calculate contour errors and a control law to mitigate them. Various techniques such as the traditional PID [12], optimal control [13], adaptive control [14], fuzzy logic control [15], iterative learning control [16], robust control [17,18] and model predictive control [19,20] have been proposed to implement CCC.

It is often not feasible to implement contour control techniques on commercial CNC machines because they do not typically allow structural modifications to their built-in controllers. Moreover, raising the controller gains of commercial CNC machines to improve tracking performance is often avoided because of the need to guarantee stability under various operating conditions. An alternative approach for reducing contour errors under such restrictive conditions is pre-compensation, where the reference position commands are modified (offline) to correct for impending contour errors before sending them to the machine [21–27]. Chin et al. [21,22] introduced a cross coupled pre-compensation technique where the desired path velocity was modified to enhance tracking accuracy. Lo and Hsiao [23,24] proposed a pre-compensation method for repetitive machining of identical parts, where they modified the reference trajectory based on the measured contour error from the first machined part. Altintas et al. [25,27] presented a vibration avoidance and contour error pre-

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* Corresponding author. Fax: 1 734 647 9379.
E-mail address: okwudire@umich.edu (C.E. Okwudire).

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compensation algorithm for feed drives by applying input shaping filters to the reference axis commands. The contour errors were estimated from the closed loop transfer functions of the drives and the shaped trajectory commands. Zhang et al. [26] presented an approach for pre-compensating contour errors in a five-axis machine where the position and orientation errors were predicted using analytically expressed tracking errors of the servo drives based on a splined toolpath. Pre-compensation techniques have been proven to be effective when servo errors can be predicted reliably and when external/unknown disturbance forces are negligible, such as in light machining (e.g., finishing), laser cutting, 3D printing and other non-contact manufacturing applications. Pre-compensation can easily be implemented in commercial CNCs using advanced functionalities (e.g., the high-speed binary program operation functionality [28]) which allow user-defined reference commands to be generated offline and fed into the CNC controllers directly.

A major problem with existing pre-compensation techniques (e.g., [23–27]) is that their performance degrades at sharp corners because they have no look-ahead feature. Moreover, they have no mechanism to ensure that kinematic constraints of the machine are not violated after pre-compensation. These shortcomings are addressed in this paper by proposing a pre-compensation approach based on a model predictive control (MPC) framework [29]. The pre-compensation input at each time step is defined as a weighted average of contour errors over a prediction horizon. Then a search algorithm is used to determine an optimal set of weights that minimize an objective function, defined as the sum of squares of the estimated contour errors over each prediction horizon, subject to machine kinematic constraints.

This paper is organized as follows. Section 2 presents an overview of the traditional approach for servo contour error pre-compensation, followed in Section 3 by a detailed explanation of the proposed MPC-based pre-compensation technique. In Section 4, numerical simulations and on-machine experiments are used to demonstrate significant improvements in contouring performance by the proposed pre-compensation technique, in comparison with the method recently proposed by Zhang et al. [26], which follows the traditional approach for pre-compensation.

2. Overview of servo contour error pre-compensation

Consider, for the sake of simplicity, a biaxial servo system whose block diagram is shown in Fig. 1(a). The desired (or reference) position is given by $p_d = [x_d, y_d]^T$, while the actual position is defined as $p_a = [x_a, y_a]^T$. Due to the limited bandwidth of the x and y servo controllers, p_a will deviate from p_d as depicted in Fig. 1(b). Let us define the contour position $p_c = [x_c, y_c]^T$ as a point on the desired trajectory which has the shortest distance to the actual position p_a. The uncompensated contour error $\varepsilon = [\varepsilon_x, \varepsilon_y]^T$ is the distance between p_a and p_c at any given point along the desired trajectory. The goal of pre-compensation is to add a control input (i.e., the pre-compensation amount) $u = [u_x, u_y]^T$ to p_d such that the resulting modified position reference command $\tilde{p}_d = [\tilde{x}_d, \tilde{y}_d]^T = p_d + u$ reduces ε after pre-compensation.

The pre-compensation amount u is a function of ε which, in general, is a nonlinear function of the axis tracking errors $e = [e_x, e_y]^T = p_d - p_a$ [1]. In model-based pre-compensation approaches, p_a is determined from a model of the servo system of Fig. 1(a), assumed here to be a linear time-invariant (LTI) system described by the discrete-time state equations

$$z(k+1) = \mathbf{A}z(k) + \mathbf{B}p_d(k) \tag{1}$$

$$p_a(k) = \mathbf{C}z(k) + \mathbf{D}p_d(k) \tag{2}$$

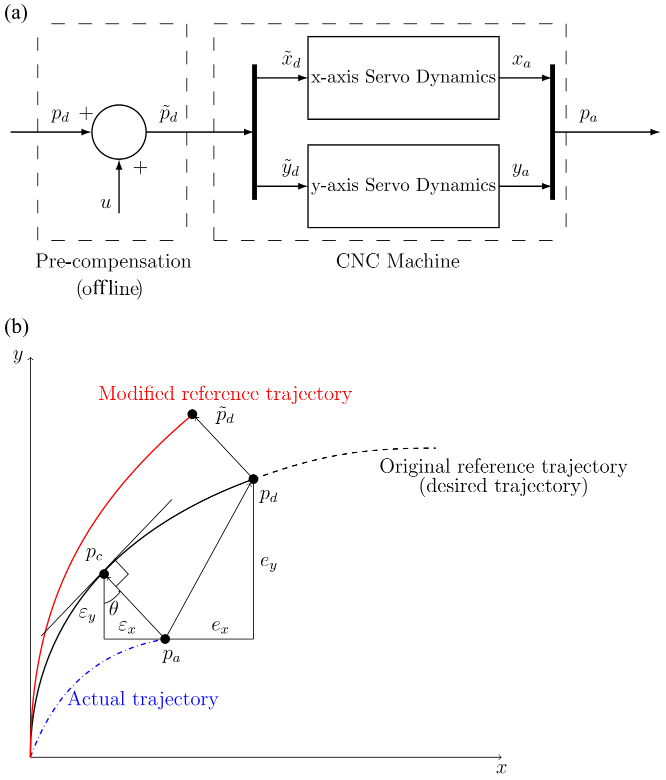

Fig. 1. (a) Block diagram of biaxial servo system with contour error pre-compensation. (b) Desired, actual and modified (i.e., pre-compensated) reference trajectories of biaxial servo system.

where $k = 0, 1, 2, \ldots, (n-1)$ are discrete time steps at a fixed sampling interval, T, and z denotes system states; while **A**, **B**, **C** and **D** are respectively the state, input, output and feed forward matrices of the biaxial servo system, respectively. Various approaches have been proposed in the literature for estimating contour errors as a function of axis tracking errors (e.g., [5,19,20,26,30,31]). In this paper, the technique proposed by Yang and Altintas [5] (summarized in Appendix A) is used to estimate contour errors ε because of its superior performance compared to simpler methods (e.g., [19,20,30]), especially in regions of sharp curvature.

With ε calculated, the traditional approach for pre-compensation is to set $u = \varepsilon$ [23–27]. This approach works very well when the desired trajectory has small curvatures, but fails in at least two respects when curvatures are sharp: (1) the traditional approach is not able to anticipate the sharp changes in curvature well in advance, therefore it does a poor job of compensating contour errors; (2) sharp curvatures demand large pre-compensation amounts, u, which could cause the compensated trajectory to violate prescribed kinematic limits of the servo axes. The next section presents an improved pre-compensation approach that generates u using a model predictive control (MPC) framework in a manner that overcomes both shortcomings of the traditional pre-compensation methods.

3. Pre-compensation approach using MPC framework

Model predictive control (MPC) is a feedback control technique that uses model-based predictions of the responses of a system over a finite time interval (i.e., prediction horizon) to generate an optimal set of control signals that minimize an objective function subject to constraints [29]. It must be noted that the method presented in this paper does not employ MPC in its strict sense. For instance, it does not make use of any feedback signals and it is not designed for real time implementation like the standard MPC