Journal of Molecular Liquids 250 (2018) 80–87

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Analytical study on liquid-solid particles interaction in the presence of heat and mass transfer through a wavy channel

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ARTICLE INFO

Article history:
Received 11 September 2017
Received in revised form 15 November 2017
Accepted 20 November 2017
Available online 22 November 2017

Keywords:
Heat transfer
Electrokinetic pumping
Mass transfer
Wavy channel
Power-law model

ABSTRACT

A comprehensive study is presented on the liquid and solid particles interaction propagating through a finite symmetric wavy channel. A non-Newtonian power law model is used to determine the flow behavior with heat and mass transfer. Furthermore, a mathematical model for electro-osmotic flow with Magnetohydrodynamics effects has been presented. The mathematical model comprises on two-dimensional conservation equations for momentum, energy, concentration, continuity with Ohm's law, chemical reaction, electrokinetic body force and viscous dissipation are formulated with the help of Cartesian coordinate system. The electric field terms are formulated into electrical potential terms using Debye length approximation, Nernst–Planck equation, and Poisson-Boltzmann equation. Moreover, an approximation of long wavelength and small Reynolds number have been used to obtain the final equations for velocity, concentration, and temperature distribution. Series solutions are presented for velocity, concentration, and temperature distribution with the help of Homotopy perturbation method up to second order. The graphical results are presented against all the involved parameters. Applications for the present study involves hemodynamics flow in biomedical engineering as well as thermal electro-osmotic micro-pumps. Furthermore, the present study is also relevant to improve thermal process in microfluidics, electrophoretic process in physiology and chromatography.

1. Introduction

Nowadays, the interaction between the fluids and solid particles arises frequently in different areas of biological and physical sciences. The interaction of the fluid and solid particles may influence the viscosity and the rheological behavior of the suspension. It is well known that anisotropic and clusters particle microstructures are the outcomes due to the movement of the particles occurs by particle-wall and particle-particle interaction [1–4]. According to the technological point of view, various industrial and chemical process include particle-fluid systems. Fluidization technology mainly depends upon particle-fluid ("solid and bubbles") interactions. Pneumatic and hydraulic propagation of particulate materials includes hydrodynamics interactions between the conveying medium (gas/liquid) and a material to be propagated. Furthermore, particle-fluid interaction includes paper coatings, sewage sludge, filtration of polymer melts, thickening and sedimentation of slurries, fixed bed, slurry reactors, trickle bed and disposal of wastes in a mineral industry etc. However, some more applications of particle-fluid interaction involve the propagation of red blood cells in a capillary flow, electrophoresis, chromatographic separations and the separation of macromolecules according to their size. Mekheimer et al. [5] discussed the peristaltic motion of particles and fluid through a planar channel. They used perturbation method to obtain the series solution against the momentum equation. Usha et al. [6] addressed the stability features of particulate suspension flow propagation through a wavy channel. The peristaltic motion of particle fluid suspension through a uniform and non-uniform annulus was discussed by Mekheimer [7]. He obtained the exact solutions against the velocity distribution and discussed the pumping characteristics and friction forces along the whole annulus. Mukhopadhyay et al. [8] studied numerically the concentrated particulate-suspension through a wavy channel. They used Finite element method to determine the solutions of the nonlinear differential equations. In the recent year, different authors studied the particle-fluid suspension in various medium using different kinds of fluid models. Some more studies on the said topic are available in references [9–14].

The applications of heat and mass transfer over a particle-fluid flow are widely present in different engineering process and everyday life. In a wide variety of engineering systems, ranging from petroleum refinement to sediment, from a nuclear reactor to internal combustion engines, propagation of pollutant in aquatic environment and production of pharmaceutical to nanotechnology includes various liquids that transport different materials to another phase in the form of drops, bubbles, and particles. Furthermore, in multi-phase simulation, the

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https://doi.org/10.1016/j.molliq.2017.11.123
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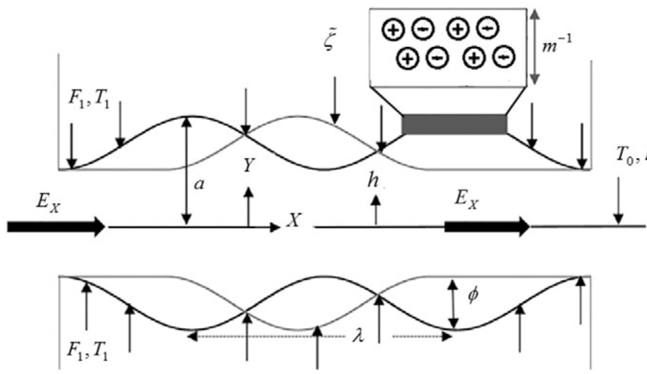

Fig. 1. Schematic diagram of Electro-osmotically modulated wavy motion between a symmetric channel at a constant temperature and concentration at the bottom and the top of a surface.

constituents have different physical features and propagates with different velocities. The constituents in a multiphase simulation i.e. particulate suspension, always interchange angular momentum, mass, energy and linear momentum. The process of mass, energy and momentum interactions are always associated in multiphase engineering systems. For instance, direct contact heat exchanger, in which the colder drops are sprayed in the center to be condensed, the drops absorb enthalpy from a vapor, and thus the temperature rises. It happens due to the direct relationship between the vapor and the cooler drops, some of the mass of vapor condenses from the drop's surface and thus enhances the average size of drops. Therefore, the hydrodynamic interaction between drops and vapor or between various drops, big drops may divide into smaller drops. Deen et al. [15] presented a numerical simulation for heat transfer flow over particle-fluid systems. Flamant et al. [16] discussed the dense suspension of solid particles with heat transfer and present a model for on-sun roof concept. Sheikholeslami and Gorji-Bandpy [17] also discussed the free convection magnetized nanofluid flow with heat transfer using control volume finite element method. Later, Hatami et al. [18] considered the motion of small spherical particles in a plane Couette flow and obtained the solution using multi-step differential transform method. Analytical study of heat and mass transfer with magnetized solid particles was discussed by Bhatti et al. [19]. Some more pertinent studies on the current topic can be found from references [20–25].

The main purpose of the present study is to determine the behavior of liquid-solid particles interaction propagating through a wavy channel in the presence of heat and mass transfer. In the present, we have considered the power law model and the special attention has been devoted to discussing the shear thinning effects. The mathematical model comprises on two-dimensional conservation equations for momentum, energy, concentration, continuity with Ohm's law, chemical reaction, electrokinetic body force and viscous dissipation are formulated with the help of Cartesian coordinate system. The electric field terms are formulated into electrical potential terms using Debye length approximation, Nernst–Planck equation, and Poisson-Boltzmann equation. The Homotopy perturbation method has been used to derive the solutions of the resulting nonlinear differential equations.

2. Formulation of problem

The geometric flow model for the electrostatically modulated wavy flow of power law fluid model through a finite symmetric channel as shown in Fig. 1.

The wall surface through a symmetric channel is described as

$$H(X,t) = a + d\cos\frac{2\pi}{\lambda}[X - ct] \tag{1}$$

The governing equations for fluid and particle phase are defined as

2.1. Fluid phase

$$\frac{\partial U_f}{\partial X} + \frac{\partial V_f}{\partial Y} = 0 \tag{2}$$

$$\begin{aligned}(1-C)\rho_f\left(\frac{\partial U_f}{\partial t} + U_f\frac{\partial U_f}{\partial X} + V_f\frac{\partial U_f}{\partial Y}\right) &= -(1-C)\frac{\partial P}{\partial X} + \mu_f(1-C)\left(\frac{\partial \xi_{xx}}{\partial X} + \frac{\partial \xi_{yx}}{\partial Y}\right) + \frac{CS'}{\omega_v}(U_p - U_f) - \sigma B_0^2 U_f \\ &\quad + \rho_e E_X\end{aligned} \tag{3}$$

$$(1-C)\rho_f\left(\frac{\partial V_f}{\partial t} + U_f\frac{\partial V_f}{\partial X} + V_f\frac{\partial V_f}{\partial Y}\right) = -(1-C)\frac{\partial P}{\partial Y} + \mu_f(1-C)\left(\frac{\partial \xi_{xy}}{\partial X} + \frac{\partial \xi_{yy}}{\partial Y}\right) + \frac{CS'}{\omega_v}(V_p - V_f) \tag{4}$$

$$(1-C)\rho_f c_p\left(\frac{\partial T_f}{\partial t} + U_f\frac{\partial T_f}{\partial X} + V_f\frac{\partial T_f}{\partial Y}\right) = K(1-C)\frac{\partial^2 T_f}{\partial Y^2} + \frac{\rho_p c_p C}{\omega_T}(T_p - T_f) + \frac{CS'}{\omega_v}(U_f - U_p)^2 - \frac{\partial q_r}{\partial Y} \tag{5}$$

$$\begin{aligned}(1-C)\left(\frac{\partial F_f}{\partial t} + U_f\frac{\partial F_f}{\partial X} + V_f\frac{\partial F_f}{\partial Y}\right) &= D_m(1-C)\frac{\partial^2 F_f}{\partial Y^2} \\ &\quad + \frac{D_m K_T}{T_m}(1-C)\frac{\partial^2 T_f}{\partial Y^2} \\ &\quad + \frac{\rho_p c_p C}{\omega_c}(F_p - F_f) - \kappa(F_f - F_0)\end{aligned} \tag{6}$$

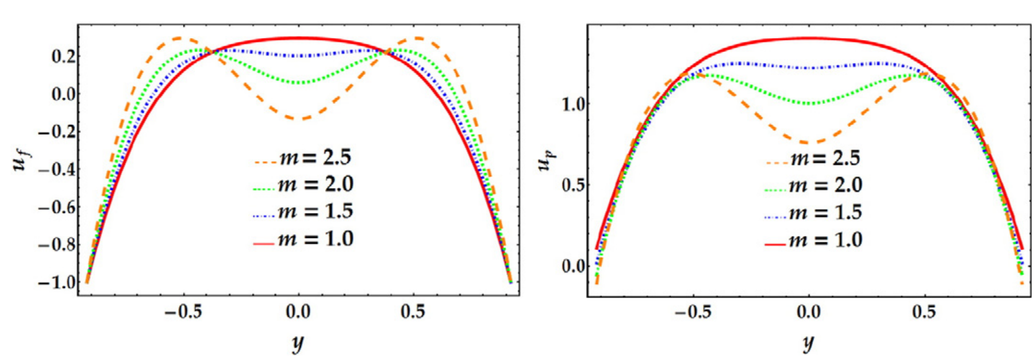

Fig. 2. Fluid phase and particulate phase velocity profile against multiple values of m.