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Damage and plastic friction in initially anisotropic quasi brittle materials

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ABSTRACT

A three dimensional micro-mechanical model is developed for modeling micro-crack growth and plastic frictional sliding in initially anisotropic quasi brittle materials under compressive loading. Macroscopic strains are attributed to elastic deformation of matrix and displacement discontinuity on micro-cracks. Effective elastic properties of cracked materials are determined using a Eshelby's solution based linear homogenization technique by considering micro-cracks as spheroidal inclusions. An efficient numerical method is used to calculate Hill polarization tensor for spheroidal micro-cracks arbitrarily embedded in transversely isotropic solid matrix. Based on this, the plastic strain related to frictional sliding on closed micro-cracks is determined by combining irreversible thermodynamics and homogenization method. A specific plastic friction criterion is formulated in terms of the local stress field on crack surfaces. The presence of a back stress tensor allows description of material hardening and softening without any additional functions. A specific damage evolution law is finally proposed. The evolutions of the friction-related plastic strain and crack-propagation induced damage are inherently coupled. A series of numerical assessments are presented for various loading paths such as uniaxial compression, triaxial compression and shear. The obtained numerical results clearly reveal that the macroscopic behaviors of cracked materials are strongly affected by the initial anisotropy. Finally, the performance of the proposed micro-mechanical model is verified by comparing numerical results and experimental data for a typical rock-like materials, shale.

1. Introduction

The induced damage affects not only mechanical properties but also other physical properties such as permeability and heat conductivity. Significant efforts have been made during the last decades on modeling of induced damage in different engineering materials. Without giving an exhaustive list, a number of macroscopic damage models have been developed in the framework of irreversible thermodynamics. In these models, spatial distribution of micro-cracks is mathematically represented by scalar or tensorial variables. Effective properties of cracked materials are determined by postulating specific forms of free energy function. Damage evolution is determined through the definition of a specific damage criterion. Directly

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calibrated from experimental data, most of such models are able to capture main features of macroscopic behaviors of brittle materials. However, these macroscopic approaches are not able to consider physical mechanisms at relevant material scales and usually meet difficulties in dealing with some specific features such as unilateral effects, damage-plastic friction coupling etc. On the other hand, for a class of quasi-brittle materials, important irreversible strains are also observed and coupled with damage evolution. By considering such irreversible strains as plastic strains, different macroscopic models have also been developed for the description of coupled elastic–plastic strain and damage evolution. Without intention to deliver a comprehensive review of a great number of plastic damage models developed so far, we just mention here some representative and recent works. Most models have been formulated in the framework works of irreversible thermodynamics by choosing suitable internal variables for damage state and plastic strains. The key step is to define an appropriate free energy or enthalpy function incorporating coupling between two dissipation mechanisms (Hansen and Schreyer, 1993; Houlsby and Puzrin, 2000; Abu Al-Rub and Darabi, 2012; Krairi and Doghri, 2014; Balieu and Kringos, 2015). Specific models have been proposed for concrete materials (Ju, 1989; Cicekli et al., 2007; Voyiadjis et al., 2008; Zhu and Sun, 2013; Nguyen et al., 2014) and for rock materials (Lubarda and Krajcinovic, 1995; Chiarelli et al., 2003; Shao et al., 2006; Lai et al., 2009; Shojaei et al., 2014; Parisio et al., 2015; Ma et al., 2016). Some models have been devoted to viscoplastic modeling of polycrystalline materials by considering brittle to ductile damage evolution (Shojaei et al., 2013). Shojaei et al. (2014) have proposed a multiscale constitutive model for anisotropic ceramic matrix composites by using the continuum damage mechanics. In most previous models, plastic strains have been described using macroscopic yield criteria and plastic potentials, depending on macroscopic stresses. The physical mechanism of macroscopic plastic strain was not explicitly explained. However, in concrete and rock like materials, it is known that macroscopic plastic strains are generally related to frictional sliding along closed micro-cracks. This microscopic mechanism will be explicitly taken into account in the present work by using a micro-mechanical approach.

Indeed, micro-mechanical approaches provide an alternative way to deal with damage-friction problems and have gained important advances in the last decades. The main objective is to consider growth and frictional sliding of micro-cracks at relevant scales and to determine effective properties of cracked materials by a up-scaling method. Different features should be considered such as induced anisotropy, interaction between cracks and unilateral effects. Two categories of micro-mechanical damage models can be found. In the first category, the formulation of micro-mechanical models is directly based on results issued from Linear Fracture Mechanics, without providing a rigorous up-scaling procedure. On the other hand, a second category of micro-mechanical models is based on rigorous linear homogenization procedures for heterogeneous materials. This kind of approach is adopted in the present work. In this context, micro-cracks are considered as ellipsoidal inclusions embedded in an elastic matrix. Effective elastic properties of cracked materials are estimated using an Eshelby solution based homogenization method, for instance (Barthélémy et al., 2003; Dormieux et al., 2006; Zhu et al., 2008, 2009, 2011). In particular (Zhu and Shao, 2015), have recently proposed a micro-mechanical model for rock-like materials with closed frictional micro-cracks by taking into account coupling between crack growth and frictional sliding. Further, some micro-mechanical models have been extended to saturated brittle materials by incorporating effects of fluid pressure (Xie et al., 2012).

However, most micro-mechanical models so far developed deal with crack growth and frictional sliding in initially isotropic solid matrix. However, for many engineering materials, there is an inherent anisotropy due to oriented microstructures such as bedding planes, preferential orientations of mineral grains or pores etc. It is therefore an important issue to consider effects of the initial structural anisotropy in plastic damage modeling. In the context of Eshelby solution based micro-mechanical approach, an essential step of homogenization procedure is to establish a suitable relation between prescribed macroscopic strain and local strain field inside crack phases using a specific strain concentration tensor. Such a concentration tensor is directly related to the so-called Eshelby tensor or equivalently Hill's polarization tensor. In the case of ellipsoidal micro-cracks embedded in an isotropic elastic solid matrix, it is possible to obtain a closed-form of Hill's tensor. However, it is no more possible to get such closed-form solutions for micro-cracks embedded in an anisotropic elastic matrix. Suitable numerical procedures have to be developed to calculate Hill's tensor. In this paper, an efficient numerical method is presented for calculating Hill's tensor for three-dimensional spheroidal micro-cracks embedded in a transversely isotropic solid matrix. This method is based on numerical integration of the exact Green function proposed by (Pan and Chou, 1976) and a frame transformation produce presented in (Giraud et al., 2007). Using this method, the micro-mechanical damage model initially proposed in (Zhu and Shao, 2015) is extended to initially anisotropic brittle materials. The proposed approach also open the perspective to deal with damage-friction problems in brittle materials with an general initial anisotropy. A series of numerical assessments are presented for different loading paths such as triaxial compression and shear. The performance of the micro-mechanical model is also verified through experimental validation for a typical shale that was extensively investigated in the context of geological disposal of radioactive waste and shale gas exploration. In particular, interactions between the initial anisotropy and induced damage anisotropy are analyzed.

2. Characterization of cracked material

Consider a representative elementary volume (REV) of cracked materials, occupying a geometric domain Ω limited by its external boundary surface $\partial\Omega$. The REV is composed of a transversely isotropic linear elastic solid and a number of oblate ellipsoidal micro-cracks, as shown in Fig. 1. All micro-cracks with the same normal vector are put into the same family, which is characterized by its normal vector $\underline{n}$, mean radius a and mean half opening c. The volume fraction per unit volume of micro-