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A smooth curve evolution approach to the feedrate planning on five-axis toolpath with geometric and kinematic constraints

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ABSTRACT

Feedrate planning with geometric and kinematic constraints is crucial for sculptured surface machining. Due to the non-linear relationship between the Cartesian space and the joint space, the feedrate planning method for a given five-axis toolpath is very limited compared with that in three-axis machining. To achieve the exact control of the chord error and the kinematic characteristics of cutter and machine tool, this paper presents a new feedrate planning method for five-axis parametric path using a smooth curve evolution strategy. The constraints in feedrate planning are first classified as two types of neighbor-independent (NI) constraints and neighbor-dependent (ND) constraints. Then for constraint violated region, the detailed formulas of determining the update feedrates of violated sampling points are given using a decoupled manner. As a result, NI and ND constraints are satisfied respectively with one step and multi-step smooth curve evolution technique, which can smoothly deform the target feedrate profile to the desired update positions. Simulations and experiments are performed on the given tool path to validate the effectiveness of the proposed feed planning method. The results show that the proposed method is robust and effective in the exact control of constraints in the feedrate planning on complex five-axis toolpath.

1. Introduction

Five-axis machine tool is widely applied to complex surface machining in the fields of power, aerospace, mold and die. For achieving a high-efficiency and high accuracy machining, a key step in five-axis NC machining is to plan feedrate along a given five-axis toolpath with or without smoothing process [1–3]. However, even if the tool path is geometrically smooth, it does not mean that a smooth cutter movement is certain to be generated. Owing to the nonlinearity of kinematic transformation between the Cartesian space and the joint space, it is extremely possible to generate significant feed fluctuations of tool center point (TCP) especially under condition of high speed machining. Therefore, with a view to ensure the machining accuracy and machining quality, an interpolator with rotation tool center point (RTCP) function, which has the ability of accurate control of cutter tip, has become a research focus. RTCP interpolator usually defines the five-axis toolpath in part coordinate system. After performing feedrate interpolation, an inverse kinematics transformation is carried out in the NC unit and consequently the TCP coordinates are transformed to the actual position of each machine axis. Up to now, it still is a crucial issue to plan smooth feedrate with geometric and kinematic constraints for five-axis NC machining.

Different from three-axis machining, five-axis machining involves simultaneous translational movements and rotational movements, the coupling effects between two types of movements lead to that some solutions of feedrate planning for three-axis machining cannot be directly extended to five-axis machining. For example, if the feedrate profile of cutter tip is generated based only on *S* curve Acc/Dec control, it is not ensured that the accelerations and jerks of all drive axes are within the allowable ranges in five-axis machining. Although the solutions to feedrate planning can be achieved using different methods and respecting different constraints, limited methods are currently available for five-axis machining. No matter on-line or off-line mode, the goal of feedrate planning is basically to find a desired feedrate profile along a fixed toolpath with predefined constraints. Generally, besides discrete cutter location data format, continuous five-axis toolpath is often expressed as dual-NURBS curve [4] or a combination of position spline and orientation spline [5]. Constraints in on-line feedrate planning for these types of toolpath are those that are single, deterministic or can be got by recursive subdivision of path parameter. For example, Xu et al. [6] proposed an angular velocity limited feedrate interpolation method for bi-parameter curves. Fatan and Feng [7] presented a method to access the geometry-based errors for interpolated tool paths in five-axis

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surface machining. Qiao et al. [8] proposed a fast linearization method of dual NURBS curves with adaptive nonlinear error control for major five-axis machine tools.

If more constraints are respected in the feedrate planning for five-axis machining, it is usually performed in off-line mode. In order to take as an indicator for trajectory optimization by highlighting the zones for which the programmed feedrate is not reached, Lavernhe et al. [9] proposed a prediction model of kinematical performances for the feedrate planning in five-axis machining. aiming at constant feed interpolator for five-axis machining, Fleisig and Spence [10] presented a reduced angular acceleration interpolation algorithm for off-line interpolation of a set of discrete cutter location points by means of near arc-length parameterized splines, which include the position, orientation and reparametrization splines. Li et al. [11] presented a NURBS pre-interpolator with the converter function of linear/circular segments for a CNC system, so that the NURBS interpolator can be thoroughly applied for five-axis machining. More recently, in order to confine the axis accelerations and jerks, the feedrate optimization modes have respectively established using linear programming and sequential quadratic programming methods [12,13], from which the feedrate profile of cutter tip can be directly obtained once the concrete machine configuration is given in advance. Based on the 5-axis toolpath represented by two B-spline curves, Beudaert et al. [14,15] built a 5-axis corner rounding model to smooth the tool path geometry, which is suitable for acceleration and jerk limited feedrate interpolation. Further, for a given 5-axis NURBS toolpath an iterative velocity profile optimization method was proposed for axis jerk limit. Yuen et al. [16] presented a spline interpolation technique for five-axis machining of sculptured surfaces. The tool tip locations and cuter axis orientation are modeled as quintic splines independently to achieve geometric jerk continuity while respecting the relative changes in position and orientation of the cutter along the curved path.

As aforementioned, the feedrate planning for five-axis toolpath has brought new challenges. Based on our previous study [17], this paper presents a smooth feedrate planning method for five-axis toolpath with geometric and kinematics constraints. Different from constrained optimization techniques, our method is based on a smooth curve evolution strategy with a decoupling manner for preventing high-frequency feedrate fluctuation. Besides constraints of chord error and axis accelerations and jerks, the kinematic characteristics of cutter can also be constrained for ensuring good machining quality. The remainder of this paper is organized as follows. Section 2 describes five-axis toolpath and interpolation algorithm. The proposed smooth curve evolution strategy for feedrate planning is given in Section 3. Section 4 gives the detailed implementation of feedrate planning. Simulations and experiments are conducted in Section 5 to validate the proposed method. Section 6 concludes the paper.

2. Five-axis tool path and interpolation algorithm

A five-axis machining toolpath can be described by a dual-parametric curve with following expressions [4]:

$$\boldsymbol{C}(u) = \sum_{i=0}^{n} \boldsymbol{C}_i N_{i,k}(u),\ \ \boldsymbol{B}(u) = \sum_{i=0}^{n} \boldsymbol{B}_i N_{i,k}(u),\ \ u \in [0,1] \tag{1}$$

where $n+1$ is the number of control points and the knot sequence represents the variation of the parameter u along the tool path. k is the degree and is often set as 3. $\boldsymbol{C}_i\boldsymbol{C}_i$ and $\boldsymbol{B}_i$ represent the control points, u is the normalized arc length parameter of the cutter tip curve $\boldsymbol{C}(u)$. Both Bspline curves $\boldsymbol{C}(u)$ and $\boldsymbol{B}(u)$ have the same parameter setting in order to synchronize the two curves. $N_{i,k}(u)$ is the B-spline basis function with the following recursive formulas

$$\boldsymbol{N}_{i,k}(0) = \begin{Bmatrix} 1 & u_i \le u \le u_{i+1} \\ 0 & \text{others} \end{Bmatrix}$$

$$\boldsymbol{N}_{i,k}(u) = \frac{u-u_i}{u_{i+k}-u_i}\boldsymbol{N}_{i,k-1}(u) + \frac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}}\boldsymbol{N}_{i+1,k-1}(u)$$

From Eq. (2) the unit vector $\boldsymbol{O}(u)$ of cutter orientation at path parameter u is calculated as

$$\boldsymbol{O}(u) = [\boldsymbol{B}(u) - \boldsymbol{C}(u)]/\|\boldsymbol{B}(u) - \boldsymbol{C}(u)\| \tag{2}$$

Then, for CL data $[\boldsymbol{P}(u)\,\boldsymbol{O}(u)]^T$, its corresponding drive displacement $\boldsymbol{m}(u) = [X(u), Y(u), Z(u), \Phi(u), \Psi(u)]^T$ of a given machine tool is also a function of path parameter u, where $[X\ Y\ Z]$ and $[\Phi\ \Psi]$ represent the positions of linear and rotary axes respectively. Owning to different machine configurations, the specific mathematical expressions of axis displacements may be different. They generally can be expressed as

$$\begin{cases} \Phi(u) = m_1(\boldsymbol{O}(u)), \quad \Psi(u) = m_2(\boldsymbol{O}(u)) \\ X(u) = m_3(\boldsymbol{C}(u), \boldsymbol{O}(u)),\ Y(u) = m_4(\boldsymbol{C}(u), \boldsymbol{O}(u)) \\ Z(u) = m_5(\boldsymbol{C}(u), \boldsymbol{O}(u)) \end{cases} \tag{3}$$

If a feedrate $f(u)$ is given along the tool tip curve $\boldsymbol{C}(u)$, then by applying the second-order Taylor expansion formula a commonly used feedrate interpolation algorithm can be given as

$$u_{i+1} = u_i + \frac{f(u_i)T}{\|\boldsymbol{C_u}(u_i)\|} + \left(\frac{a(u_i)}{\|\boldsymbol{C_u}(u_i)\|} + \frac{\boldsymbol{C_u}(u_i)\cdot\boldsymbol{C_{uu}}(u_i)f^2(u_i)}{\|\boldsymbol{C}`(u_i)\|^4}\right)\frac{T^2}{2} \tag{4}$$

where $a(u_i)$ is the acceleration of feedrate. It indicates that the determination of sampling positions mainly depends on the geometric characteristics of cutter tip curve $\boldsymbol{C}(u)$ and the predefined feedrate profile $f(u)$ for the position control at regular time interval T in a real-time five axis CNC interpolator.

3. Smooth curve evolution strategy for feed planning

In order to generate the feedrate profile along a given five axis dual-parametric toolpath, a smooth curve evolution based feedrate planning module is developed as a pre-processor, which is able to incorporate more constraints and reduce the computational burden in real-time parametric curve interpolator or in adaptive machining with varying feedrate. The architecture of the proposed curve evolution based feedrate planning method for five-axis machining is shown in Fig. 1. The advantages of curve evolution based feedrate planning method is that constraints are more easily incorporated or relaxed instead of conventional optimization method, and at the same time the proposed method is able to keep the whole feedrate profile more smooth. For a given cutter tip curve $\boldsymbol{C}(u)$, a NURBS is used to define the feedrate profile as a function of path parameter u

$$f(u) = \sum_{i=0}^{e} d_i\omega_i N_{i,k}(u) / \sum_{i=0}^{n} \omega_i N_{i,k}(u) \tag{5}$$

where d_i is the control point of the NURBS curve, and $e+1$ is the number of control points. ω_i is the weight factor, and k is the degree of the NURBS spline.

In order to achieve a high-quality and high-efficiency machining, it is necessary in feedrate planning to control the geometric error and the kinematic characteristics of cutter and drive axes within their own preset ranges while keeping the feedrate as high as possible. In the proposed feedrate planning strategy, the