International Journal of Plasticity 56 (2014) 232–261

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

An improved damage evolution law based on continuum damage mechanics and its dependence on both stress triaxiality and the third invariant

L. Malcher [*], E.N. Mamiya

Department of Mechanical Engineering Faculty of Technology, University of Brasília, Brasília, DF, Brazil

ARTICLE INFO

Article history:
Received 27 September 2013
Received in final revised form 17 December 2013
Available online 18 January 2014

Keywords:
Calibration point
Ductile fracture
Stress triaxiality
Normalized third invariant

ABSTRACT

In this contribution, it is suggested a damage evolution law which is based on Continuum Damage Mechanics (CDM) and dependent on the hydrostatic pressure, by the stress triaxiality, and the third invariant of deviatoric stress tensor, by the so-called normalized third invariant. The contribution has been motivated meanly by the reason that the accuracy in describe the mechanical behavior of materials and the predictive fracture onset ability of damage constitutive models are strongly dependent on the loading condition used to procedure the calibration of material parameters. Regarding classical damage models as Lemaitre and Gurson, the level of material degradation can be optimist or conservative for loading conditions far from the calibration point. In the first part of this paper, the suggested damage evolution law is presented and the new state and dissipation potential are determined. The plastic flow rules for associative and non-associative plasticity are derived and an implicit numerical integration algorithm is suggested, based on the operator split methodology. The numerical algorithm is also implemented in an "in house" finite element framework and its robustness is tested for a set of numerical simulations upon wide range of stress triaxiality. Numerical results are compared with experimental data presented in literature and parameters as reaction curve, evolution of the equivalent plastic strain and damage variable at fracture are analyzed. In a critical situation, the numerical results have shown that the original damage models as Lemaitre's model has a prediction of 68% in disagreement with experimental data and the proposed damage evolution law has around 1%, regarding the determinations of the displacement at fracture initiation.

1. Introduction

The correct prediction of fracture in ductile materials has become, in recent years, a subject of great importance for the competitive sectors of industry, such as automotive, aerospace, marine, and military. For example, the weight reduction of vehicle structures, such as chassis and bodies, without a loss of performance, has promoted advanced yield criterion for mechanical projects that allow the determination of the correct displacement at fracture and the site of fracture initiation. Thus, the design of new products requires careful planning of each step necessary for its development, optimization and manufacturing. Since the late sixties, numerous mathematical models have been formulated to satisfactorily describe the mechanical behavior of macro and microscopic ductile fractures in metallic materials. The following models are the most popular models found in the literature: McClintock (1968) developed a model that assumes the emptiness inside of a metal

---

[*] Corresponding author.
E-mail address: malcher@unb.br (L. Malcher).

http://dx.doi.org/10.1016/j.ijplas.2014.01.002
0749-6419/

matrix is in the form of a cylinder. The Rice and Tracey (1969) model considers a void to be a perfect sphere. The Gurson–Tvergaard–Needleman model (1977 and 1984) describes the elastic–plastic behavior of porous materials (Gurson, 1977). The Lemaitre model (see Lemaitre, 1985; Lemaitre and Chaboche, 1990) uses the principles of Continuum Damage Mechanics (CDM). The Oyane (1980), Cockcroft and Latham (1968) and Johnson and Cook (1985) models use experimental observations to describe the fracture.

The formulations proposed by Lemaitre and Gurson are the most important coupled damage models for ductile materials, regarding the Continuum Damage Mechanics (CDM) and the Micromechanics of defects (see Chaboche et al., 2006). Since then, motivated by the limitations of these classical models, such as in the prediction of the correct fracture location or in the determination of the correct values of the internal variables at fracture, many researchers have proposed improvements to both methodologies by introducing more effects in the constitutive formulation or in the damage evolution law, such as the effects of pressure and temperature, the Lode angle dependence, visco-plastic effects, the crack closure effect, and shear mechanisms (Tvergaard and Needleman, 1984; Rousselier, 1980, 2001; Barsoum and Faleskoh, 2007a,b; Xue, 2007; Nahshon and Hutchinson, 2008; Nielsen and Tvergaard, 2009; Tvergaard, 2008; Lemaitre and Chaboche, 1990; Chaboche, 2003; Andrade Pires et al., 2003; Chaboche et al., 2006; Besson, 2010; Mirone and Corallo, 2010; Li et al., 2011; Stoughton and Yoon, 2011; Khan and Liu, 2012; Malcher et al., 2014).

In general, these classical coupled damage models have the ability to predict the correct fracture location under a specific stress state or range of combined stress triaxialities and Lode angle parameters (see Xue, 2007; Nahshon and Hutchinson, 2008; Teng, 2008; Malcher et al., 2012, 2013; Brunig et al., 2013; Reis et al., 2010) and are extremely accurate for determining the loading conditions close to the calibration point (see Malcher et al., 2012, 2013). For example, within a range of high levels of stress triaxialities, where spherical void growth is the predominant mechanism, the models based on Gurson theory, such as the Gurson–Tvergaard–Needleman model, have good performance in the prediction of the fracture location and the parameters in the fracture, such as the equivalent plastic strain and the displacement. However, under shear-dominated loads, where failure is mainly driven by the plastic strain shear localization of the inter-voids ligaments due to void rotation and distortion, the model does not perform well (see Engelen et al., 2005; Chaboche et al., 2006).

Based on the context presented, in this contribution, a damage evolution law is proposed that aims to improve the damage model of Lemaitre (1985) by including isotropic hardening and damage. The work begins with the study of the original model of Lemaitre through the conventional specimens that result in different levels of stress triaxiality. Based on the results of this study, we aimed to demonstrate the inaccuracy of this formulation with respect to the prediction of the correct time (displacement) and potential sites for ductile fracture initiation, when the loading condition imposed is presented away from the fixed point as the calibration of the elasto-plastic parameters and evolution of damage parameters varies, such as the hardening curve, the exponent and the denominator of damage. After this preliminary analysis, the objective is to adjust the accuracy and the strong dependence on the calibration point of the Lemaitre model. To perform the modification, a function is developed called the “denominator of damage function”, instead of the denominator of damage, which was originally presented as a material constant. Regarding this modification, a new state potential and dissipation potential for the new model is defined, thus maintaining the thermodynamic consistency of the formulation. A new evolution law for the damage variable is then determined, as well as other internal variables, such as plastic deformation and a variable associated with isotropic hardening. A new numerical integration algorithm is suggested for the proposed formulation, based on the operator split methodology (Simo and Hughes, 1998), and new numerical simulations are performed to demonstrate the predictive ability of the new formulation to determine the correct displacement at fracture, as well as the correct potential sites for crack initiation.

2. Theoretical aspects

In this section mathematical formulation related to Lemaitre’s original model are presented as well as the definition of some importants elasto-plastic parameters, such as: stress triaxiality and Lode angle. After that, regarding the original Lemaitre’s damage evolution law, the effect of the denominator of damage is demonstrated over the determination of the correct displacement at fracture for ductile materials. A new damage evolution law is also proposed, upon the introduction of the effect of both the stress triaxiality and third invariant like a function called “denominator of damage function”.

2.1. Lemaitre’s original model

The proposition of this paper starts from the original damage model of Lemaitre (see Lemaitre, 1985; Lemaitre and Chaboche, 1990). According to this damage model, the Helmholtz free energy is taken as the thermodynamic potential or state potential and is defined as a function of the following set of internal variables, $(\boldsymbol{\varepsilon}^e, r, \boldsymbol{\beta}, D)$:

$$\psi = \psi^{ed}(\boldsymbol{\varepsilon}^e, D) + \psi^{p}(r, \boldsymbol{\beta}) = \frac{1}{2}\boldsymbol{\varepsilon}^e : (1 - D)\mathbb{D} : \boldsymbol{\varepsilon}^e + \bar{\rho}\psi^{I}(r) + \frac{a}{2}\boldsymbol{\beta} : \boldsymbol{\beta}, \tag{1}$$

where $\mathbb{D}$ is the standard elasticity tensor, ψ represents the thermodynamic potential, and $\psi^{ed}(\boldsymbol{\varepsilon}^e, D)$ and $\psi^{p}(r, \boldsymbol{\beta})$ are the elastic-damage and plasticity contributions in the free energy, respectively. The terms $\boldsymbol{\varepsilon}^e, r, \boldsymbol{\beta}$ and D represent the elastic strain tensor, the internal variable associated with isotropic hardening, the internal variable associated with kinematic hardening and the isotropic damage variable, respectively. Regarding the thermodynamic internal variable model (Lemaitre and Chaboche, 1990), the thermodynamic forces associated with the internal variables can be determined as: