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# A visco-hyperelastic damage model for cyclic stress-softening, hysteresis and permanent set in rubber using the network alteration theory

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ABSTRACT

The large deformation time-dependent mechanical response of rubber-like materials under cyclic loading is characterized by stress-softening, hysteresis and permanent set. To describe this set of first-order phenomena a constitutive model integrating the physics of polymer chains and their alteration under cyclic loading is proposed. The time-dependency is considered using a Zener-type framework in which the chain extensibility limit is described with both physical and phenomenological approaches. The efficiency of the proposed constitutive model is illustrated by comparisons with experimental data obtained on a styrene–butadiene rubber submitted to cyclic tension loading up to failure both in constant and variable amplitudes.



## 1. Introduction

The prediction of the mechanical behavior of rubber-like materials subjected to cyclic loading has been an important research issue for this last two decades. In fact, the behavior of rubber-like materials is hyperelastic and time-dependent, and they undergo a substantial stress-softening during the first few load cycles at a given strain level. This phenomenon is often referred to as the Mullins effect; it is observed in addition to the hysteresis phenomenon (i.e. a difference between unloading and reloading paths) which is caused by the viscous effects. After the first few cycles, the material response becomes repeatable, and just before failure, an increased deterioration in the mechanical properties is noticed (Ayoub et al., 2011a). In addition to these phenomena, it could be observed that a permanent set (also known as the residual strain for which the stress is zero at unloading) appears after the first cycle and remains relatively constant under cyclic loading. There have been numerous experimental and theoretical studies dealing with the different characteristics of elastomeric behavior under cyclic loading: Mullins effect (Mullins, 1948, 1969; Mullins and Tobin, 1957; Bueche, 1960, 1961; Harwood et al., 1967; Simo, 1987; Govindjee and Simo, 1991; Johnson and Beatty, 1993; Wineman and Huntley, 1994; Miehe, 1995; Ogden and Roxburgh, 1999; Beatty and Krishnaswamy, 2000; Marckmann et al., 2002; Qi and Boyce, 2004; Chagnon et al., 2004, 2006), hysteresis (Green and Tobolsky, 1946; de Gennes, 1971; Lubliner, 1985; Doi and Edwards, 1986; Reese and Govindjee, 1998; Septanika and Ernst, 1998; Bergstrom and Boyce, 1998, 2000; Besdo and Ihlemann, 2003; Laiarinandrasana et al., 2003; Amin et al., 2006; Tomita et al., 2008; Ayoub et al., 2010a, 2011a; Freund et al., 2011; Shim and Mohr, 2011), continuous stress-softening (Ayoub et al., 2011a; Drozdov, 2012; Drozdov et al., 2013), and failure (Mars and Fatemi, 2002; Le Cam et al., 2004; Verron

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and Andriyana, 2008; Ayoub et al., 2010b, 2011c, 2012). However, currently there is no constitutive model that captures the combined effect of this entire set of phenomena.

Accordingly, the aim of this paper is to describe the total behavior of rubber-like materials subjected to cyclic loading until failure, i.e. the Mullins effect, the continuous stress-softening, the hysteresis and the residual strain amount. The developed model is based on a previous work of Ayoub et al. (2011a) in which samples of styrene–butadiene rubber (SBR) were preloaded until the maximum elongation encountered by the material during cyclic loading in order to avoid taking into account the Mullins effect and the permanent set.

The present paper is organized as follows. We first outline in Section 2 the elements of the visco-hyperelastic constitutive equations. In Section 3, the network alteration kinetics is identified on experimental observations of cyclic fatigue tests carried out on SBR until failure. In Section 4, the developed model is compared with the experimental data corresponding to constant and variable amplitude loading conditions. Finally, remarks and conclusions are given in Section 5.

Throughout the paper, vector and tensor quantities are represented with bold-face symbols, while scalars and individual components of vectors and tensors are written in italics.

## 2. Visco-hyperelastic constitutive equations

A Zener-type model (a nonlinear spring in parallel with a nonlinear Maxwell element) is used to represent the visco-hyperelastic response of elastomeric materials. The equilibrium path (i.e. the time independent part of the stress–strain response) is captured by the nonlinear spring (branch *A* in Fig. 1) while the time-dependent deviation from the equilibrium state is captured by the nonlinear Maxwell element (branch *B* in Fig. 1) itself composed of a nonlinear spring in series with a viscous dashpot.

### 2.1. Summary of the kinematics

A key quantity in the finite strain kinematic framework is the deformation gradient $\mathbf{F} = \nabla\mathbf{x}(\mathbf{X}, t)$ mapping a material point from its initial position $\mathbf{X}$ in the reference configuration $A_0$ to its actual position $\mathbf{x}$ in the current configuration $A_t$. The Zener-type model being constructed with the Taylor assumption, the deformation gradients within both parts, $\mathbf{F}_A$ and $\mathbf{F}_B$, are identical to the total deformation gradient $\mathbf{F}$:

$$\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B \tag{1}$$

The total Cauchy stress tensor $\mathbf{T}$ in the elastomeric material is given as the tensorial sum of the network Cauchy stresses $\mathbf{T}_A$ and $\mathbf{T}_B$:

$$\mathbf{T} = \mathbf{T}_A + \mathbf{T}_B \tag{2}$$

The deformation gradient $\mathbf{F}_B$ in the Maxwell branch *B* is multiplicatively decomposed into an elastic (network orientation) part $\mathbf{F}_B^N$ and a viscous (flow) part $\mathbf{F}_B^f$ as stated by the inelasticity theory (Lee, 1969; Sidoroff, 1974; Lubliner, 1985):

$$\mathbf{F}_B = \mathbf{F}_B^N \mathbf{F}_B^f \tag{3}$$

The time derivative of the total, elastic and viscous deformation gradients can be also introduced:

$$\dot{\mathbf{F}}_B = \mathbf{L}_B\mathbf{F}_B, \quad \dot{\mathbf{F}}_B^N = \mathbf{L}_B^N\mathbf{F}_B^N \text{ and } \dot{\mathbf{F}}_B^f = \mathbf{L}_B^f\mathbf{F}_B^f \tag{4}$$

where $\mathbf{L}_B = \nabla\mathbf{v}$ is the gradient of the spatial velocity $\mathbf{v} = \dot{\mathbf{x}}\mathbf{x}^{-1}$:

$$\mathbf{L}_B = \mathbf{L}_B^N + \mathbf{F}_B^N\mathbf{L}_B^f\mathbf{F}_B^{N^{-1}} \tag{5}$$

With no loss in generality (Boyce et al., 1989), it is assumed that the viscous spin is zero. The viscous deformation gradient $\mathbf{F}_B^f$ is then obtained by integrating the following equation:

$$\dot{\mathbf{F}}_B^f = \mathbf{F}_B^{N^{-1}}\mathbf{D}_B^f\mathbf{F}_B \tag{6}$$

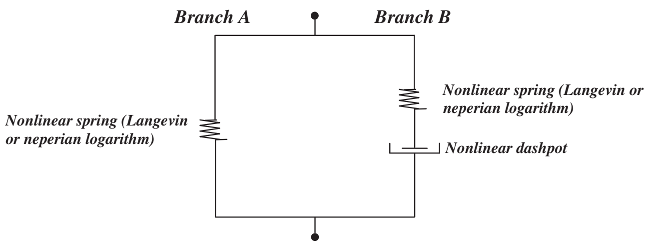


**Fig. 1.** Zener-rheological model.