International Journal of Plasticity 54 (2014) 34–55

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

Logarithmic stress rate based constitutive model for cyclic loading in finite plasticity

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ARTICLE INFO

Article history:
Received 13 May 2013
Received in final revised form 9 August 2013
Available online 22 August 2013

Keywords:
Finite plasticity
Cyclic loading
Logarithmic stress rate
Ratchetting

ABSTRACT

Based on the logarithmic stress rate, a constitutive model is developed to describe the material behaviour under cyclic loading histories (including ratchetting) in the framework of finite plasticity by using combined nonlinear isotropic and kinematic hardening rules. The nonlinear kinematic hardening rule is extended from that developed by Abdel-Karim and Ohno (2000) for infinitesimal plasticity. The cyclic hardening/softening feature of materials is reflected by using a nonlinear isotropic hardening rule. Then, the proposed model is implemented into a finite element code (e.g., ABAQUS) by employing a simple fully-implicit time-integration procedure. Finally, some numerical examples are carried out to verify the capability of the model to predict the cyclic deformation of materials in finite deformation by comparing the predictions with the corresponding experiment results in referable literature. The predicted stress responses during a simple shear with large shear strain and ratchetting during the cyclic loading tests in finite deformation are in good agreement with the corresponding experimental results.

1. Introduction

Ratchetting, a cyclic inelastic deformation accumulation induced by stress-controlled cyclic loading with non-zero mean stress, plays an important role in the safety assessment and fatigue life estimation of engineering components (Kang, 2008). The ratchetting has been extensively studied in the last decades, but still keeps being one of the most popular topics for cyclic plasticity. In the framework of small deformations, large number of constitutive models have been developed to describe the uniaxial and multiaxial ratchetting of various materials in terms of corresponding experiments as reviewed by Kang (2008) and Chaboche (2008), and more recently by Abdel-Karim (2009, 2010), Colak (2008), Hassan et al. (2008), Rahman et al. (2008), Razaiee-Pajand and Sinaie (2009), Bai and Chen (2009), Chen et al. (2009), Kang et al. (2009), Krishna et al. (2009), Lim et al. (2009), Guo et al. (2011), Guo et al. (2012), Chaboche et al. (2012, 2013), Kim et al. (2012), Yu et al. (2012a,b), Taleb (2012), Darabi et al. (2012), Xiao et al. (2012) and Pham et al. (2013), etc. Since it is well-known that the nonlinear kinematic hardening rule can catch the main feature of ratchetting evolution, the above-referred constitutive models are mainly obtained by extending the Armstrong–Frederick kinematic hardening model (Armstrong and Frederick, 1966) in various aspects.

However, it should be noted that in some cases, finite plasticity occurs during the ratchetting of materials. For example, Khan et al. (2007) performed cyclic deformation tests (including ratchetting) of OFHC copper under biaxial tension–torsion cyclic loading with relatively large shear strain. Kang et al. (2006, 2009) investigated the whole-life ratchetting and

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http://dx.doi.org/10.1016/j.ijplas.2013.08.004

ratchetting-fatigue interaction of SS304 stainless steel and annealed 42CrMo steel, and obtained the ratchetting evolution of the materials with ratchetting in the range of finite strain, i.e., up to 40%. It requires a constitutive model constructed in the framework of finite plasticity for cyclic loading. Recently, much effort was done to extend the nonlinear Armstrong–Frederick kinematic hardening model to describe the strain hardening of materials in the range of finite deformation, such as done by Tsakmakis (1996), Tsakmakis and Willuweit (2004), Svendsen et al. (1998), Lion (2000), Mollica et al. (2001), Shen (2006), Shutov and Kreißig (2008), Dettmer and Reese (2004), Vladimirov et al. (2008), Henann and Anand (2009), Anand (2011) and Anand et al. (2012), etc. There are two kinds of procedures to construct the kinematic hardening rules for finite plasticity approaches. One is realized by using the hyper-elasticity theory proposed by Lion (2000), in which the inelastic part obtained in the standard Kröner multiplicative decomposition (1960) of finite deformation can be further multiplicatively decomposed into two parts, i.e., energetic and dissipative parts. It requires employing strain-like internal variables to construct the kinematic hardening rules and establishing corresponding evolution equations in the same configuration of the body. The other is formulated by using the rate-type hypo-elasticity theory, in which a stress-like internal variable is introduced to construct the kinematic hardening rule, and the stretching tensor **D** is additively decomposed into two parts, i.e., elastic $\mathbf{D}^e$ and plastic $\mathbf{D}^p$. This approach requires adopting an appropriate objective stress rate to establish the evolution equations with frame-indifference. For the first procedure, additional strain-like variables will make constitutive equations very complicated and inconvenient for numerical implementation; while for the second, there is no such problem. In the rate-type finite plasticity approaches, one of the key issues is the choice on an adequate objective stress rate. Much effort has been devoted to investigating objective stress rates so far. Several objective stress rates have been introduced into the finite plasticity approaches, such as the Zaremba–Jaumann–Noll rate (Zaremba, 1903;Jaumann, 1911; Noll, 1955;Thomas, 1955; Prager, 1960), Oldroyd rate (Oldroyd, 1950), Cotter–Rivlin rate (Cotter and Rivlin, 1955), Truesdell rate (Truesdell, 1965), Green–Naghdi–Dienes rate (Green and Naghdi, 1965;Dienes, 1979, 1987), Durban–Baruch rate (Durban and Baruch, 1977), Sowerby–Chu rate (Sowerby and Chu, 1984), Xia–Ellyin rate (Xia and Ellyin, 1993), logarithmic rate (Xiao et al., 1997a; Xiao et al., 1997b; Bruhns et al., 1999), etc. Xiao et al. (1997a, 1998), Bruhns et al. (1999, 2001a,b, 2005), and Meyers et al. (2003, 2006) proved that the rate-type models are self-consistent with the notion of elasticity and are furnished by simple, natural conditions with integrability if and only if the logarithmic rate was adopted. Otherwise, some unreasonable responses, such as shear oscillatory and elastic stress ratchetting, may be observed when the finite elastic behaviour was predicted by the finite deformation models developed from the hypoelasticity approach. In such models, the rate-type stress–strain equation is directly extended from the elastic stress–strain one in small deformation.

Therefore, in this paper, based on the logarithmic stress rate, an elasto–plastic constitutive model is developed in the framework of rate-type finite plasticity for cyclic loadings, and the nonlinear kinematic hardening rule is constructed from extending the Ohno–Abdel-Karim rule which is proposed by Abdel-Karim and Ohno (2000) in the range of small deformation. Also, a nonlinear isotropic hardening rule is used to reflect the cyclic hardening/softening feature. The proposed model is then implemented into a finite element code (ABAQUS) by using a simple, fully-implicit time-integration procedure based on the radial return and backward Euler integration methods. Finally, the capability of the proposed model to predict the cyclic deformation of materials in the range of finite deformation is verified by comparing the predictions with corresponding experimental results from the literature (Ishikawa, 1999;Khan et al., 2007; Kang et al., 2006).

2. Finite elasto–plastic constitutive model for cyclic loading

2.1. Kinematics

Considering a homogeneous deformable body $\mathfrak{B}$, and assuming that **X** is an arbitrary position vector of a material particle in $\mathfrak{B}$ in a fixed reference configuration and **x** is the corresponding space position vector denoted by $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$ at current time, the deformation gradient, velocity and velocity gradient are written, respectively, as

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \quad \mathbf{v} = \dot{\mathbf{x}}, \quad \mathbf{L} = \frac{\partial \dot{\mathbf{x}}}{\partial \mathbf{X}} = \dot{\mathbf{F}}\mathbf{F}^{-1}. \tag{1}$$

Hereafter, the symbol $(\dot{\cdot})$ denotes the material time derivative of scalar or tensor fields.

The well-known left polar decomposition of **F** is given by

$$\mathbf{F} = \mathbf{VR}, \quad \mathbf{R}^{\mathrm{T}} = \mathbf{R}^{-1}, \quad \mathbf{B} = \mathbf{V}^2 = \mathbf{FF}^{\mathrm{T}}, \tag{2}$$

where the orthogonal tensor **R** is a rotation tensor; while the symmetric positive definite tensors **V** and **B** are left stretch tensor and left Cauchy–Green tensor, respectively.

Generally, the symmetric part of **L** is defined as stretching tensor, denoted by **D**, and its skew part is defined as vorticity tensor, denoted by **W**. It yields

$$\mathbf{L} = \mathbf{D} + \mathbf{W}, \quad \mathbf{D} = \frac{1}{2}\left(\mathbf{L} + \mathbf{L}^{\mathrm{T}}\right), \quad \mathbf{W} = \frac{1}{2}\left(\mathbf{L} - \mathbf{L}^{\mathrm{T}}\right). \tag{3}$$