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Short Communication

# Identification of dynamic instabilities in machining process using the approximate entropy method

Daniel Pérez-Canales [a], José Álvarez-Ramírez *,[b], Juan Carlos Jáuregui-Correa [c], Luciano Vela-Martínez [d], Gilberto Herrera-Ruiz [a]

[a] Facultad de Ingeniería, Universidad Autónoma de Querétaro, Cerro de las Campanas s/n, Ciudad Universitaria, Querétaro, Qro., 76010, Mexico
[b] División de Ciencias Básicas de Ingeniería, Universidad Autónoma Metropolitana Iztapalapa, Apartado Postal 55-534, México D.F. 09340, Mexico
[c] CIATEQ, A.C. Unidad El Marques, Av. Manantiales s/n, Parque Industrial Bernardo Quintana, El Marques, Querétaro, Mexico
[d] dCIATEQ, A.C. Unidad Aguascalientes, Circuito Aguascalientes Norte. 135, Parque Industrial del Valle de Aguascalientes, 20355 Aguascalientes, Aguascalientes, Mexico

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ABSTRACT

Chatter instability is a persistent problem in machining process that produces vibrations characterized by nonlinear and nonstationary dynamics. Although traditional fast Fourier transform approaches are typically used for the monitoring of chattering in industry, the method is suitable only for linear and stationary signals. In this paper, methods based on approximate entropy (ApEn) are proposed to identify chatter instabilities in milling process. as entropy is an index of the irregularity and complexity related to randomness from data series. The attractiveness of the ApEn approach is that it can deal with nonlinear and nonstationary data, requires a relatively small number of observations and can be used for noisy signals. For a lab-scale milling experimental setup, the ApEn was implemented under a time–frequency monitoring method, showing that instable chattering is associated with entropy increment for a frequency range. In contrast, stable milling led to an entropy pattern where high-entropy values are concentrated at high frequencies, which are related to the natural dynamics of the cutting tool.


## 1. Introduction

Nowadays, machining processes in industry demand higher productivity with smaller tolerances. Modern machining processes involve increased material removal per unit time, higher spindle speeds, increased feed rate and greater depth of cut. However, process instabilities can appear at certain combination of machining parameters, resulting in decreased accuracy, poorer surface finish, reduced tool life and even worst-case spindle failure [1]. A distinctive feature of dynamic instabilities in machining processes is nonlinear and nonstationary behavior [2]. A clear example is the chatter phenomenon, which is one of the most problematic issues in machining process. Chattering signals are quite complex, usually noisy and nonstationary [3]. An approach for understanding and modeling chatter instabilities is to model the process and theoretically identify the principal dynamic components inducing chattering instabilities. While modeling of linear systems is well established, modeling of nonlinear systems is usually a quite complicated task. An alternative to modeling is to deal directly with signals obtained from the measurement of physical variables involved in the process. Examples of these physical variables are acceleration, vibration, acoustic emission, cutting forces and driver current which can be measured through a great variety of sensors. The measured signals are processed in different ways in order to find features that can provide important information of the process. Traditional monitoring methods use fast Fourier transform (FFT) as a signal processing method. Although FFT methods are theoretically constrained to linear stationary processes, acceptable results have been obtained in practice for a great variety of manufacturing processes [4].

Although traditional FFT analysis can provide an acceptable detection of the principal spectral components involved in an unstable chatter phenomenon, novel approaches derived from nonlinear methods has been explored to gain further insights in the nature and complexity of this kind of signals. Examples of these methods are wavelet transform and Hilbert–Huang transform (HHT) [5–8], which can provide temporal information of the dominant spectral components involved in the process as well as of potential nonlinearities that can lead to unstable dynamics. A shortcoming of wavelet is that it is sensitive to the specified basis functions, so their implementation requires wide experience to interpret the wavelet output (e.g., coherence). On the other hand, many other approaches can deal with nonlinear and nonstationary signals are based on nonlinear parameter estimation methods that are related to chaos theory. These approaches

* Corresponding author.
*E-mail address:* jjar@xanum.uam.mx (J. Álvarez-Ramírez).

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have been used to identify chatter instabilities from signals as they can take the signal as a time series and extract important process information. The idea of parameter estimation techniques is that time series are not commonly a very compact representation of a time evolving phenomenon, so that it is necessary to condense the information and focus on finding parameters that contains the most relevant features of the underlying system [9]. Parameters from such nonlinear methods related to chaos theory are measures of dimension, rate of generated information (entropy) and Lyapunov spectrum [10].

In this paper, a methodology known as approximate entropy (ApEn) is proposed to identify chatter instabilities from acceleration signals. ApEn is a method proposed by Pincus [10] to measure system irregularity (complexity) from data series. ApEn no longer focuses on identifying the attractor of the process but on estimating irregularity in contrast to well-known nonlinear parameter estimation methods such as correlation dimension [11] and K-S entropy [12]. This is an advantage of ApEn since it requires orders of magnitude fewer points for its calculations [13]. ApEn can be applied to noisy and medium data sets which are undefined of infinitive for K-S entropy and correlation dimension [14].

ApEn measures the logarithmic frequency with which similar blocks of certain length remain similar for blocks whose length is augmented by one position with a similarity tolerance. As a result, a low ApEn value is associated with a strong regularity. In contrast, a high ApEn value implies irregularity (complexity) related to randomness [15]. ApEn can potentially distinguish a great variety of systems: low-dimensional deterministic systems, periodic and multiply periodic systems, high-dimensional chaotic systems, stochastic and mixed (stochastic and deterministic) systems. As mentioned previously, the ApEn approach requires fewer data points for its calculation with good reproducibility, and is nearly unaffected by noise whose magnitude is below a specified filter level [14]. ApEn has been extensively applied for biomedical diagnosis [16–18]. More recently, ApEn was applied to financial time series [19,20] and mechanical systems [21]. Of particular interest for the aims of this work are the results in [21], that showed the ability of ApEn to characterize machine operation conditions, confirming a relation between the regularity level of the acceleration signal and the health state of a machine. It was demonstrated that ApEn is a good index to quantify the degree of instability in machine tools for a wide range of sampling frequencies and for a relatively small number of observations.

The characteristics of the chatter phenomenon [21], involving nonlinear complex signals, make suitable the ApEn to monitor instabilities in machining processes. To this end, lab-scale milling experiments were performed at different operating conditions and the stability of the measured acceleration was monitored within a time–frequency framework of the ApEn method. In this way, it is expected that stability–instability changes can be detected by changes of the ApEn values at certain frequency ranges. The results showed that an increment in the regularity of the signal indicated by a higher ApEn value is related to an instability increment of the process.

## 2. Methodology

The entropy $H(X)$ of a single discrete random variable $X$ is a measure of its average uncertainty. In statistical mechanics, entropy is essentially a measure of the number of ways in which a system may be arranged, often taken to be a measure of "disorder" (the higher the entropy, the higher the disorder). This definition describes the entropy as a measure of the number of possible microscopic configurations of the individual atoms and molecules of the system (microstates) which would give rise to the observed macroscopic state (macrostate) of the system. Traditional methods quantify the degree of uncertainty of a time series by evaluating the appearance of repetitive patterns. For instance, the Shannon's entropy is calculated by the equation

$$H(X) = -\sum_{x_j \in \Theta} p(x_i)\log(p(x_i)) = -E[\log(p(x_i)]$$

where $X$ represents a random variable with a set of values $\Theta$ and probability mass function $p(x_i)=P_r\{X=x_i\}$, $x_i \in \Theta$, and $E$ represents the expectation operator. It is noted that $p\log(p) \to 0$ if $p \to 0$. The Shannon-entropy characterizes the gain of information and measure disorder and uncertainty. The relevance of this concepts relies on the fact that entropy measures the exponential rate at which information is obtained.

### 2.1. Approximate entropy

A drawback of traditional entropy concepts is that computations require an infinite data series with infinitely accurate precision and resolution. However, experimental data are commonly short and noisy. To deal with this problem, Pincus [10] introduced the approximate entropy (ApEn) statistic to quantify regularity and complexity in real time series. The approximate entropy computations are based on the likelihood that templates in the time series that are similar remain similar on next incremental comparisons. Time series with large approximate entropy must have substantial fluctuation of irregularity. Given its simplicity from both conceptual and algorithmic standpoints, ApEn has found sound application in many science and engineering fields, including physiology, geophysics and financial systems.

The ApEn computation algorithm can be described as follows. Consider a finite time series of length $N$ sampled at small time intervals $T_s$, $\{X_i\}=\{x_1,x_2\ldots,x_N\}$, where $x_i=x(t_0+iT_s)$. The length $N$ defines a time scale $\tau = NT_s$. Introduce two $m$-dimensional sequence vectors $u^{(m)}(i) = \{x_i,x_{i+1},\ldots,x_{i+m-1}\}$ and $v^{(m)}(j) = \{x_j,x_{j+1},\ldots, x_{j+m-1}\}$, $i \neq j$, $1 \leq i,j \leq N-m+1$. Two vectors $u^{(m)}(i)$ and $v^{(m)}(j)$ are called similar if their distance

$$d_{u,v}(i,j) = \max\{|u(i+k)-v(j+k)| : 0 \leq k \leq m-1\} \quad (1)$$

is smaller than a specified tolerance $\varepsilon$. For each of the $N-m+1$ vectors $u^{(m)}(i)$ the number of similar vectors $v^{(m)}(j)$ is determined by measuring their respective distances. Let $n_i^{(m)}$ be the number of vectors $v^{(m)}(j)$ similar to $u^{(m)}(i)$. The relative frequency to find a vector $v^{(m)}(j)$ which is similar to $u^{(m)}(i)$ within a tolerance level $\varepsilon$ is given by

$$C_i(m,\varepsilon,\tau) = \frac{n_i^{(m)}}{N-m} \quad (2)$$

where $N-m$ is the number of vectors $v^{(m)}(j) \neq u^{(m)}(i)$ that are potentially similar to $u^{(m)}(i)$. In this way, one looks at the relative frequency of the logarithm of $C_i^{(m)}(m,\varepsilon,\tau)$, i.e.,

$$\Phi^{(m)}(m,\varepsilon,\tau) = \frac{1}{N-m+1}\sum_{i=1}^{N-m+1} \ln C_i(m,\varepsilon,\tau) \quad (3)$$

The $(m,\varepsilon,\tau)$-entropy per unit time is defined as

$$h(m,\varepsilon) = \lim_{\tau \to \infty} \frac{1}{T_s}[\Phi(m,\varepsilon,\tau) - \Phi(m,\varepsilon,\tau)] \quad (4)$$

It is noticed that $h(m,\varepsilon)$ gives the decay rate of the probabilities to find paths of length $m$ within a distance $\varepsilon$ from a typical path $X_m=\{x_1,x_2,\ldots,x_m\}$:

$$P(m,\varepsilon) \sim \exp[-mT_s h(m,\varepsilon)]$$

For finite $N$, it is estimated by the approximate entropy statistics

$$ApEn(m,\varepsilon,\tau) = \frac{1}{T_s}[\Phi^{(m)}(m,\varepsilon,\tau) - \Phi^{(m+1)}(m,\varepsilon,\tau)] \quad (5)$$