International Journal of Plasticity 26 (2010) 887–904

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

Orthotropic strain rate potential for the description of anisotropy in tension and compression of metals

Oana Cazacu [a,*], Ioan. R. Ionescu [b], Jeong Whan Yoon [c]

[a] Department of Mechanical and Aerospace Engineering, University of Florida, REEF, 1350 N. Poquito Road, Shalimar, FL 32579, USA
[b] LPMTM, University Paris 13, 99 Av. J-B. Clement Villetaneuse, France
[c] Alcoa Technical Center, 100 Technical Dr., Alcoa Center, PA 15069-0001, USA

ARTICLE INFO

Article history:
Received 7 August 2009
Received in final revised form 21 October 2009
Available online 16 November 2009

Keywords:
Plastic anisotropy
Tension-compression asymmetry
Hexagonal closed packed metals
Strain rate potential

ABSTRACT

In this paper, a macroscopic anisotropic strain rate potential, which can describe both the anisotropy and tension-compression asymmetry of the plastic response of textured metals is derived. This strain rate potential is the exact work-conjugate of the anisotropic stress potential CPB06 of Cazacu et al. (2006). Application of the developed strain rate potential to HCP high-purity alpha-titanium is presented.

1. Introduction

Currently, forming of hexagonal metals poses tremendous challenges due to their low ductility at room temperature and their unusual deformation characteristics: strong asymmetry between tensile and compressive behavior, and a very pronounced anisotropy. Traditional explanations for this unusual behavior are that, unlike cubic metals, hexagonal metals deform also by twinning. In contrast to slip, twinning is a directional shear mechanism: in general, shear in one direction can produce twinning while shear in the opposite direction cannot. For example, in magnesium alloys sheets, twinning is not active in tension along any direction in the plane of the sheet, but is easily activated in compression. As a result the average in-plane compressive yield stress is about half the average in-plane tensile yield stress (e.g. Lou et al., 2007). Thus, the yield surfaces are not symmetric with respect to the stress free condition. Since hexagonal metals sheets exhibit strong basal textures, a pronounced anisotropy in yielding is observed. Despite recent progress in crystal plasticity, modeling of hcp materials incorporating twinning and its effects on texture evolution has remained a challenge. Due to the lack of adequate macroscopic criteria, finite-element (FE) simulations of forming of hexagonal metals are still performed using classic anisotropic formulations for cubic metals such as Hill (1948) (see for e.g. Takuda et al., 1999; Kuwabara et al., 2001). The major difficulty encountered in formulating analytic expressions for the yield functions of hexagonal metals is related to the description of the tension-compression asymmetry associated to twinning. Recently, yield functions in the three-dimensional stress space that describe both the tension-compression asymmetry and the anisotropy have been developed. To describe yielding asymmetry that results either from twinning or from non-Schmidt effects at single crystal level, Cazacu and Barlat (2004) have proposed an isotropic criterion expressed in terms of all invariants of the stress deviator. This isotropic yield criterion was applied to the description of crystal plasticity results of Hosford and Allen (1973) and that of Vitek et al.

* Corresponding author. Tel.: +1 850 833 9350; fax: +1 850 833 9366.
E-mail address: cazacu@reef.ufl.edu (O. Cazacu).

0749-6419/$ - see front matter
doi:10.1016/j.ijplas.2009.11.005

(2004) for randomly oriented polycrystals (for more details, see Cazacu and Barlat (2008)). An orthotropic extension of this criterion, using the generalized invariants approach of Cazacu and Barlat (2003), has also been developed.

Cazacu et al. (2006) criterion accounts for anisotropy and yielding asymmetry between tension and compression, which are associated with deformation twinning. It was shown that this orthotropic criterion, denoted in the following as CPB06, describes with accuracy yielding of hexagonal metals (e.g. Cazacu et al., 2006; Khan et al., 2007; Graff et al., 2007; Plunkett et al., 2006, 2008; Cazacu and Barlat, 2008; Khan et al., 2009, etc.).

Ziegler (1977) has shown that, based on the plastic work equivalence principle, a strain rate potential can be associated to any convex stress potential. Hence, a strain rate potential can be also used instead of the classical stress potential to describe the plastic response of the material. The plastic response of metals can be assessed by means of micro-mechanical crystal plasticity calculations, considering the material as a collection of grains of different orientations, subjected to a given loading path. Note that it is easier to numerically obtain a crystallographic strain rate potential than to compute the crystallographic yield surface. Further, the crystallographic strain rate potential can be approximated with analytic expressions. Such an approach was applied to polycrystals with cubic crystal structure (e.g. Arminjon et al., 1994; Van Houtte, 1994; Hiwatashi et al., 1997; Van Bael and Van Houtte, 2002, 2004; Rabahallah et al., 2009, etc.). These strain rate potentials have also been used in finite-element simulations of forming operations for both FCC and BCC polycrystals (e.g. Yoon et al., 1995; Bacroix and Gilormini, 1995; Hu et al., 1998; etc.).

It is worth noting that analytic expressions of the exact strain rate potentials associated to macroscopic stress potentials are known only for the classical yield criteria such as von Mises, Tresca, Drucker-Prager, the anisotropic Hill (1948), and a special case of Hill's (1979). For non-quadratic stress potentials, obtaining an analytic expression for the exact dual is very challenging, if not impossible. Barlat and co-workers have proposed several analytic non-quadratic anisotropic strain rate potentials (Barlat and Chung, 1993; Barlat et al., 1993; Chung et al., 2000; Kim et al., 2003, 2008;). Although none of these strain rate potentials are strictly dual (conjugate) of the respective non-quadratic stress potentials, it was shown that these strain rate formulations lead to a description of the plastic anisotropy of FCC metals of comparable accuracy to that obtained by the stress potentials (see Kim et al., 2007).

In general, strain rate potentials are more suitable for process design, especially for the optimization of the initial blank shape for the purpose of reducing the earing percentage in cup drawing (see Chung et al., 2000). In summary, existing strain rate potentials (phenomenological or texture-based) are applicable only to the description of the plastic behavior of materials with cubic crystal structure (BCC or FCC). To the best of our knowledge, a strain rate formulation for hexagonal materials has not been proposed yet.

In this paper a full 3D anisotropic strain rate potential for materials that display tension-compression asymmetry is deduced (5 independent components for incompressible plasticity). It will be shown that this strain rate potential is the exact dual of the quadratic form of the anisotropic CPB06 stress potential. The approach used for deriving the strain rate potential consists in developing an exact dual for the isotropic form of the CPB06 potential and then extending this isotropic strain rate potential such as to account for orthotropic symmetry. Since the developed anisotropic strain rate potential is an exact dual of the CPB06 stress potential, the anisotropy coefficients are the same as for the stress potential. Thus, the anisotropy coefficients can be determined using either the stress based formulation or the strain based formulation in conjunction with mechanical data.

The structure of the paper is as follows. We begin with a brief presentation of the modeling framework and the analytic strain rate potentials associated with von Mises and Hill (1948) yield criteria. Further, the anisotropic CPB06 yield criterion is recalled (Section 2). The expression of the exact dual of the isotropic CPB06 stress potential is derived in Section 3. The main result of this study, an anisotropic strain rate potential for metals displaying tension-compression asymmetry is presented in Section 4. It is shown that this strain rate potential, which is the exact dual of the anisotropic CPB06 stress potential can be obtained by simply substituting in the expression of the isotropic strain rate potential the plastic strain rate tensor $\mathbf{D}^p$ by the modified strain rate tensor $\mathbf{B} = \mathbf{H}:\mathbf{D}^p$, where ":" denotes the doubly contracted product of any two tensors and $\mathbf{H}$ is a fourth-order orthotropic tensor associated with the material's plastic anisotropy. The developed anisotropic strain rate potential is applied to the description of a hcp high-purity alpha-titanium material (Section 5).

2. Strain rate potentials for description of plastic behavior

The onset of plastic flow is generally described by specifying a convex yield function, $\varphi(\sigma)$, in the stress space and associated flow rule

$$\mathbf{D}^p = \dot{\lambda}\frac{\partial\varphi}{\partial\boldsymbol{\sigma}} \tag{1}$$

where σ is the Cauchy stress tensor, $\mathbf{D}^p$ denotes the plastic strain rate tensor and $\dot{\lambda} \geqslant 0$ stands for the plastic multiplier. The yield surface is defined as $\varphi(\sigma) = \tau$, where τ is a positive scalar with the dimension of stress. Generally, τ is taken as the uniaxial yield stress in tension, σ_T. Let denote by $\mathbb{C}$ the convex domain delimited by the yield surface

$$\mathbb{C} = \{\boldsymbol{\sigma} | \varphi(\boldsymbol{\sigma}) \le \tau\}.$$

The dual potential of the stress potential $\varphi(\sigma)$ is defined (see Ziegler, 1977; Hill, 1987; Chung and Richmond, 1993, etc.) as