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A unified stability prediction method for milling process with multiple delays

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A B S T R A C T

A unified method for predicting the stability lobes of milling process with multiple delays is presented. The characteristics of delays in milling are analyzed by considering the effects of the runout and the pitch angles of the cutter. The cutter is divided into a finite number of axial elements so that the contributions of different delays and the influence of the helix angle can be considered in the governing equation. The stability lobes are obtained through the following steps. First, transform the infinite time domain into certain time discretization intervals. Second, an explicit relation between the current time interval and the previous time interval is obtained based on the governing equation. Third, a transition matrix related to every discretized time interval is constructed with the aid of the above relation. Finally, according to Floquet theory, the chatter-free axial depth of cut is derived from the eigenvalues of the transition matrix. Both numerical and experimental tests demonstrate that the proposed method is effective for milling process with multiple delays, whether with runout or with variable pitch angles. The proposed method is also applied to examine the asymptotic stability trends for different cutting condition parameters such as radial immersions, feed directions, feeds per tooth and helix angles when cutter runout occurs. Some new phenomena for certain combinations of parameters are shown and explained. It is found that feed per tooth has great effect on the stability lobes when cutter runout occurs.

1. Introduction

Chatter is a form of self-excited vibration due to the dynamic interaction between the cutter and the workpiece. The occurrence of chatter vibration leads to poor surface finish and may damage the machine spindle or the cutter in the worst case. In practice, to achieve high material removal rate and high machining quality, the milling process must be conducted in stable state. Thus, the problem arises about how to evaluate whether the selected cutting parameters will lead to stable milling or not. The basis is to develop suitable dynamic model that can reflect the chatter mechanism in milling. Many research efforts have been focused on this issue.

Altintas [1–6] is one of the pioneers in studying the dynamic behavior of the milling process. Research results in Refs. [1–6] pointed out that when the cutting forces create a relative displacement between the cutter and the workpiece at the cutting point, the chip thickness experiences waves on the inner and outer surfaces due to present and past vibrations. The gain and the phase shift between the inner and outer waves may lead to exponential growing chips and hence very large forces until the cutter jumps out of the cut [3]. The above phenomenon is the well-known chip-regenerative chatter. From this basic physical understanding, it can be found that there exist delayed position variables which could be used to couple the cutting forces to the cutter motion. The mathematical models developed to explain these phenomena correspond to delay differential equations (DDEs). Based on this important discovery, extensive efforts have been carried out to model the dynamic milling process and to develop the stability lobe diagrams that can distinguish chatter-free operations from unstable operations [1–24].

In early decades, Koenigsberger and Tlusty [7] used orthogonal cutting model to analyze the milling stability. Later, Altintas and co-workers [1–6] developed a stability method which leads to analytical determination of stability lobes directly in the frequency domain. This method, known as zero-order approximation, can achieve reasonably accurate predictions for processes where the cutting forces vary relatively small [3,24], e.g., the case of large radial immersions. However, if the process is highly intermittent, e.g., the case of small radial immersion, zero-order approximation will lead to unacceptable result [3,24]. To improve the prediction accuracy in this case, a higher-order solution was suggested to predict the stability by Budak and Altintas [8,9]. Alternatively, Tlusty and Ismail [10] pointed out that the time domain simulation would be a good choice for accurate stability predictions in milling. The closed form for expressing the dynamics of milling system in time domain was developed by

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Nomenclature

N	total tooth number of the cutter
T	spindle rotation period of the cutter
β	helix angle of the cutter, degree
f_z	feed rate of the cutter, mm/tooth
D_c	diameter of the cutter, mm
ρ,λ	cutter runout parameters: ρ is the cutter axis offset; λ is the locating angle for the offset, which is the angle between the direction of the offset and the nearest tooth (tooth 1)
τ	tooth passing period for cutter with constant pitch and $\tau=T/N$
m_x,k_x,c_x	modal effective mass, stiffness and damping coefficients in the X-direction
m_y,k_y,c_y	modal effective mass, stiffness and damping coefficients in the Y-direction
R_r,R_z	radial and axial depth of cut
Ω	spindle rotation speed
R	nominal radius of the cutter
θ	angular position of the cutter
F_T,F_R	tangential and radial cutting forces
θ_{en},θ_{ex}	entry and exit angles
i	index related to the tooth number
$\tau_{unpitch-i}$	tooth passing period related to the ith tooth of a nonconstant pitch cutter
ϕ_i	space angle between tooth i and tooth i+1 of a nonconstant pitch cutter
k	number of intervals in T
K_t,K_r	tangential and radial cutting force coefficients
M	maximum number of delays that may occur

Sridhar et al. [11], but its analytical solution was firstly successfully obtained by Minis et al. [12], who used the famous Floquet theory [13,14]. Zhao and Balachandran [15] numerically determined the stability boundary using the time domain simulations. Later, Mann and co-workers [16,17] proposed a temporal finite element analysis for solving the delayed equations written in the form of a state space model whereas Insperger and Stepan [18] employed the semi-discretization scheme to solve the stability in discrete time domain. The other research efforts mainly extended the application of the above principal solutions in frequency and time domain. For example, Merdol and Altintas [19] planned the milling process by using the stability results in frequency domain. Patel et al. [20] implemented the temporal finite element method to detect the island phenomena in milling. Note that these islands were found first by Zatarain et al. [21]. Tois [22] developed a probabilistic algorithm for a robust analysis of stability in milling based on the basic principles of semi-discretization scheme. Campomanes and Altintas [23] proposed an improved time domain simulation method for analyzing the stability at small radial immersions. Gradisek et al. [24] investigated the stability boundaries for variable radial immersions by both zero-order method and semi-discretization method.

It is worth noting that the above works were conducted under the assumption that there is only one delay term. The delay is often assumed to be constant and has the value of the tooth passing period. This is true for ideal milling operations with constant pitch cutter. However, in practice, the time delay is basically determined by the rotation of the cutter but it is also affected by the current and the delayed position of the cutting edge. That is, the time delay may be state-dependent. Insperger et al. [25,26] analyzed such state dependent delays for turning process. As to milling, due to the effect of cutter runout or unpitched space angles, the appearance of state dependent delays directly leads to the existence of multiple delayed terms.

- The occurrence of cutter runout will lead to the following explicit phenomenon. That is, the current cutting point on the inner surface wave may be to remove the outer surface wave generated by its lth previous tooth. Here, l may be more than one. As a result, multiple delays may occur. Detailed analysis of this influence is described in Section 2.2.
- The adoption of variable pitch cutter will also lead to variable delays. If cutter runout does not occur, any cutting point is always to remove the surface left by its first previous tooth. In this case, the delay term corresponding to the cutting points on the same tooth is the same. However, due to the unevenly pitched space angles, the delay corresponding to different tooth will be different. If cutter runout occurs, the delay will be influenced by both the cutter runout and the space angles. In summary, there might also exist more than one delay in this case.

Insperger et al. [27] systematically studied the frequency characteristic for evenly pitched milling cutter with runout in time domain. Their results showed that that cutter runout will shift the frequency content of the cutting force signal away from the tooth passing frequency and towards the spindle rotation frequency. The principal period of runout-milling process is equal to the spindle rotation period. Although this understanding was achieved, their research was still carried out under the assumption that the delay can be approximated as the tooth passing period. This hypothesis will have good prediction accuracy when the runout is relatively small. If the cutter runout is large, this may lose some accuracy for the actual process. On the other hand, Altintas et al. [28] and Budak [29,30] studied the stability of variable pitch milling cutter in frequency domain. Their works showed that the pitch angles have great effect on the stability boundary. Sims et al. [31] proposed a time-averaged semi-discretization method to study the chatter stability for variable pitch and variable helix milling cutters. However, the above methods [28–30] were not suitable for the case when cutter runout occurs.

A general form for multiple and distributed delays was presented by Insperger and Stepan [32]. It is worth noting that Insperger's work [32] did not study the stability problem for milling process which may have multiple delays. To have a unified model, this paper presents a new method to determine the stability lobe for the milling process with multiple delays in time domain. This method is an improved version of the semi-discretization method [18,32]. The advantage of the proposed method lies in that the algorithms performed in instantaneous consideration of every possible delay can be applied for a great range of actual milling process. It can be used to study the stability of milling process both with and without cutter runout. It is also suitable for the milling cutter either with constant or nonconstant pitch angles. Comparisons of numerical simulation and experiment results will be provided for different types of cutters and different cutting conditions to show the validity.

On the other hand, a review of the literature shows that although some research efforts have focused on the dynamic behaviour of milling processes under different cutting conditions